Name ____ Date ___________

Chapter 1 Functions and Their Graphs

Section 1.1 Lines in the Plane

Objective: In this lesson you learned how to find and use the slope of a line to write and graph linear equations.

Important Vocabulary	Define each term or concept.
Slope	
Parallel	
Perpendicular	

I. The Slope of a Line (Pages 3–4)

What you should learn
How to find the slopes of lines

The formula for the **slope** of a line passing through the points (x_1, y_1) and (x_2, y_2) is $m =$ ______________________ .

To find the slope of the line through the points (– 2, 5) and (4, – 3), ____
____.

A line whose slope is positive ____________ from left to right.
A line whose slope is negative ____________ from left to right.
A line with zero slope is __________________.
A line with undefined slope is __________________.

II. The Point-Slope Form of the Equation of a Line (Pages 5–6)

What you should learn
How to write linear equations given points on lines and their slopes

The **point-slope form** of the equation of a line is

_____________________________________ .

This form of equation is best used to find the equation of a line when ____
____.

The **two-point form** of the equation of a line is

______________________________________ .

The two-point form of equation is best used to find the equation of a line when ____

____.

Example 1: Find an equation of the line having slope – 2 that passes through the point (1, 5).

The approximation method used to estimate a point between two given points is called __________________________. The approximation method used to estimate a point that does not lie between two given points is called _________________________.

A **linear function** has the form __________________. Its graph is a ____________ that has slope _________ and a y-intercept at _____________.

III. Sketching Graphs of Lines (Pages 7–8)

What you should learn
How to use slope-intercept forms of linear equations to sketch lines

The **slope-intercept form** of the equation of a line is ________________ , where m is the _______________ and the y-intercept is (_____, _____).

Example 2: Determine the slope and y-intercept of the linear equation $2x - y = 4$.

The equation of a **horizontal line** is _________. The slope of a horizontal line is ______. The y-coordinate of every point on the graph of a horizontal line is _____________.

The equation of a **vertical line** is _________. The slope of a vertical line is _____________. The x-coordinate of every point on the graph of a vertical line is _____________.

The **general form** of the equation of a line is

_____________________________________.

Every line has an equation that can be written in ____________
______________.

When a graphing utility is used to sketch a straight line, the graph of the line may not visually appear to have the slope indicated by its equation because ________________________
____.

Example 3: Use a graphing utility to graph the linear equation $2x - y = 4$ using (a) a standard viewing window, and (b) a square window.

IV. Parallel and Perpendicular Lines (Pages 9–10)

What you should learn
How to use slope to identify parallel and perpendicular lines

The relationship between the slopes of two lines that are parallel is ____.

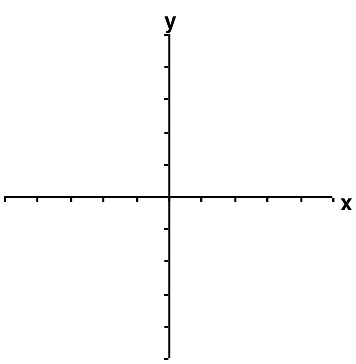
The relationship between the slopes of two lines that are perpendicular is _____________________________________
____.

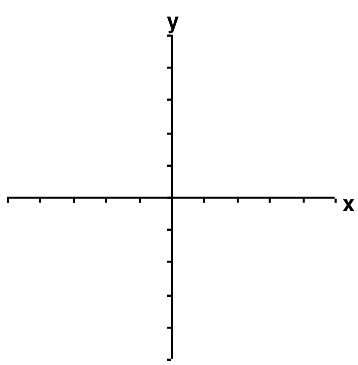
A line that is parallel to a line whose slope is 2 has slope ______.
A line that is perpendicular to a line whose slope is 2 has slope
_________.

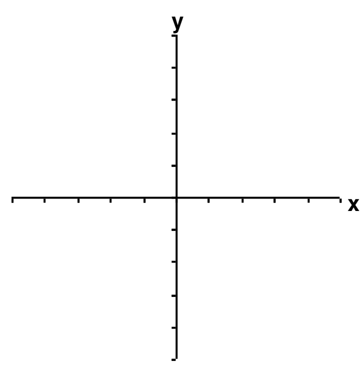
What must be done to make the graphs of two perpendicular lines appear to intersect at right angles when they are graphed using a graphing utility?

Example 4: Use a graphing utility to graph the perpendicular lines $y = 2x - 3$ and $y = -0.5x + 5$ using (a) a standard viewing window, and (b) a square window.

Additional notes







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Name ____ Date ____________

Section 1.2 Functions

Objective: In this lesson you learned how to evaluate functions and find their domains.

Important Vocabulary	Define each term or concept.
Function	
Domain	
Range	
Independent variable	
Dependent variable	

I. Introduction to Functions (Pages 16–17)

What you should learn
How to decide whether a relation between two variables represents a function

A rule of correspondence that matches quantities from one set with items from a different set is a ___________________.

In functions that can be represented by ordered pairs, the first coordinate in each ordered pair is the _______________ and the second coordinate is the _________________.

Some characteristics of a function from Set A to Set B are

1)

2)

3)

4)

To determine whether or not a relation is a function, _______

____

____________________________.

If any input value of a relation is matched with two or more output values, ____.

Some common ways to represent functions are

1)

2)

3)

4)

Example 1: Decide whether the table represents y as a function of x.

x	-3	-1	0	2	4
y	5	-12	5	3	14

II. Function Notation (Pages 18–19)

What you should learn
How to use function notation and evaluate functions

The symbol _______________ is **function notation** for the value of f at x or simply f of x. The symbol $f(x)$ corresponds to the ________________ for a given x.

Keep in mind that ____________ is the name of the function, whereas ____________ is the output value of the function at the input value x.

In function notation, the _____________ is the independent variable and the _____________ is the dependent variable.

Example 2: If $f(w) = 4w^3 - 5w^2 - 7w + 13$, describe how to find $f(-2)$.

A piecewise-defined function is ___________________________ ____.

III. The Domain of a Function (Page 20–21)

> ***What you should learn***
> How to find the domains of functions

The **implied domain** of a function defined by an algebraic expression is ____

____.

In general, the domain of a function excludes values that

____

____.

For example, the implied domain of the function $f(x) = \sqrt{5x - 8}$

is ____

____.

IV. Applications of Functions (Page 22)

> ***What you should learn***
> How to use functions to model and solve real-life problems

Example 3: The price P (in dollars) of a child's handmade sweater is given by the function $P(s) = 3s + 15$, where s represents the size (size 1, size 2, etc.) of the sweater. Use this function to find the price of a child's size 5 handmade sweater.

V. Difference Quotients (Page 23)

> ***What you should learn***
> How to evaluate difference quotients

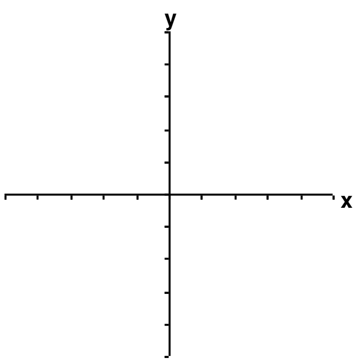
A **difference quotient** is defined as

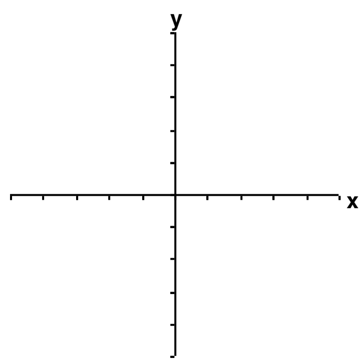
____.

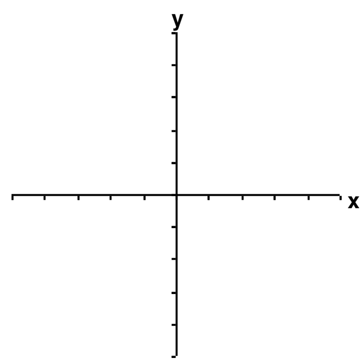
Describe a real-life situation which can be represented by a function.

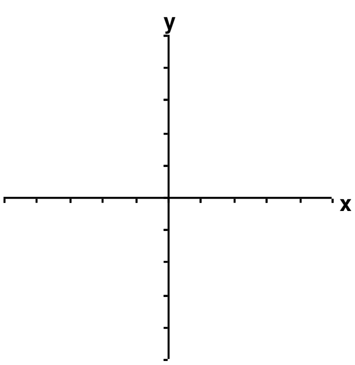
Additional notes

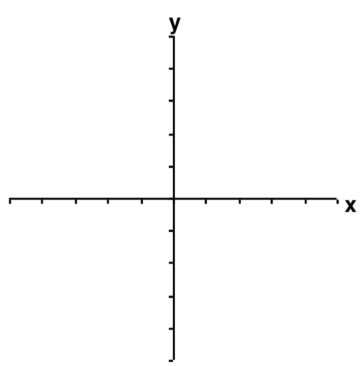
Additional notes

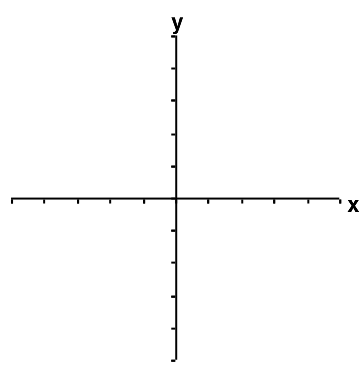












Homework Assignment

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Section 1.3 Graphs of Functions

Objective: In this lesson you learned how to analyze the graphs of functions.

Important Vocabulary Define each term or concept.

Graph of a function

Greatest integer function

Step function

Even function

Odd function

I. The Graph of a Function (Pages 29–30)

What you should learn
How to find the domains and ranges of functions and how to use the Vertical Line Test for functions

Explain the use of open or closed dots in the graphs of functions.

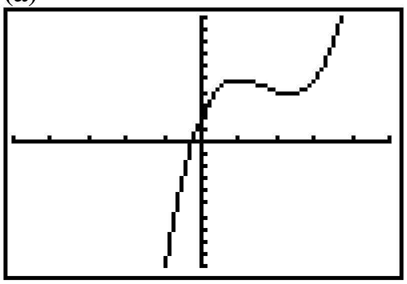
How do you find the domain of a function from its graph?

How do you find the range of a function from its graph?

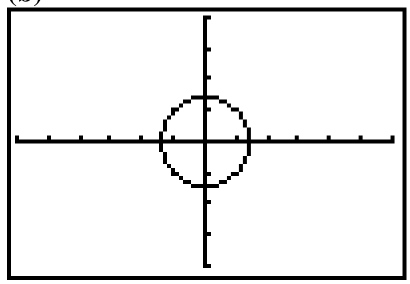
State the **Vertical Line Test** for functions.

Example 1: Decide whether each graph represents y as a function of x.

(a)



(b)



II. Increasing and Decreasing Functions (Page 31)

> ***What you should learn***
> How to determine intervals on which functions are increasing, decreasing, or constant

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval, ________________________________ .

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval, ________________________________ .

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval, ____________________________ .

Given a graph of a function, to find an interval on which the function is increasing ________________________________
____.

Given a graph of a function, to find an interval on which the function is decreasing ____________________________
____.

Given a graph of a function, to find an interval on which the function is constant ________________________________
____.

III. Relative Minimum and Maximum Values (Pages 32–33)

> ***What you should learn***
> How to determine relative maximum and relative minimum values of functions

A function value $f(a)$ is called a **relative minimum** of f if

____

____.

A function value $f(a)$ is called a **relative maximum** of f if

____

____

The point at which a function changes from increasing to decreasing is a relative ________________. The point at which a function changes from decreasing to increasing is a relative _________________.

To approximate the relative minimum or maximum of a function using a graphing utility, _________________________________

____

____

___________________________________.

Example 2: Suppose a function C represents the annual number of cases (in millions) of chicken pox reported for the year x in the United States from 1960 through 2000. Interpret the meaning of the function's minimum at (1998, 3).

IV. Step Functions and Piecewise-Defined Functions (Page 34)

What you should learn
How to identify and graph step functions and other piecewise-defined functions

Describe the graph of the greatest integer function.

Example 3: Let $f(x) = [\![x]\!]$, the greatest integer function. Find $f(3.74)$.

To sketch the graph of a piecewise-defined function, _________

____

____.

V. Even and Odd Functions (Pages 35–36)

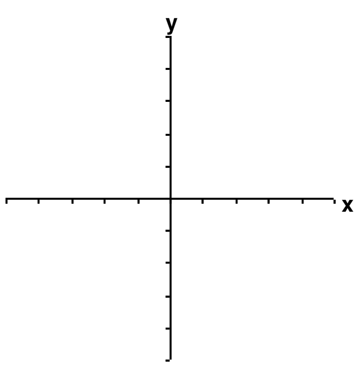
> ***What you should learn***
> How to identify even and odd functions

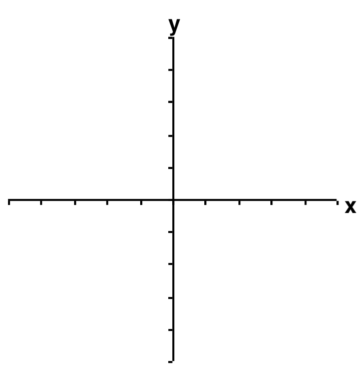
A graph is symmetric with respect to the y-axis if, whenever (x, y) is on the graph, ____________ is also on the graph. A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, ____________ is also on the graph. A graph is symmetric with respect to the origin if, whenever (x, y) is on the graph, ______________ is also on the graph. A graph that is symmetric with respect to the x-axis is

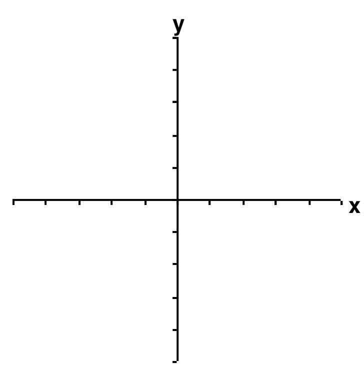
____

A function f is **even** if, for each x in the domain of f,
$f(-x) =$ ________________________.
A function f is **even** if, for each x in the domain of f,
$f(-x) =$ ________________________.

Additional notes







> **Homework Assignment**
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Section 1.4 Shifting, Reflecting, and Stretching Graphs

Objective: In this lesson you learned how to identify and graph shifts, reflections, and nonrigid transformations of functions.

Important Vocabulary Define each term or concept.

Vertical shift

Horizontal shift

Rigid transformations

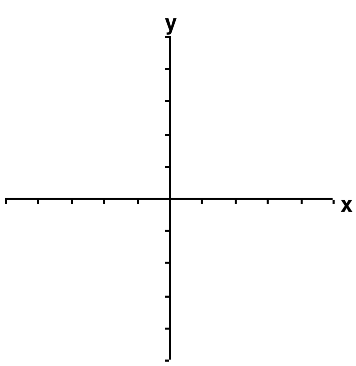
Nonrigid transformations

I. Summary of Graphs of Parent Functions (Page 41)

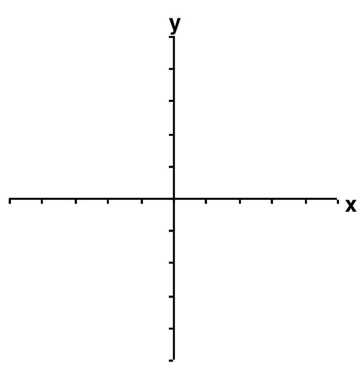
What you should learn
How to recognize graphs of parent functions

Sketch an example of each of the six most commonly used functions in algebra.

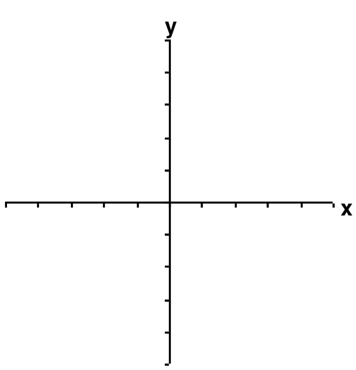
Linear Function



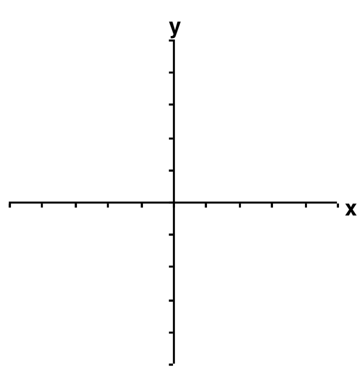
Absolute Value Function



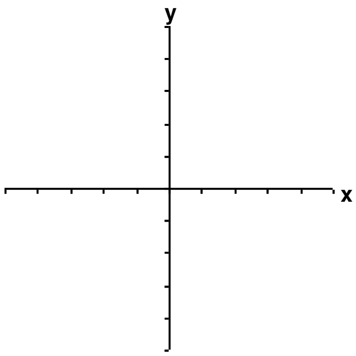
Square Root Function



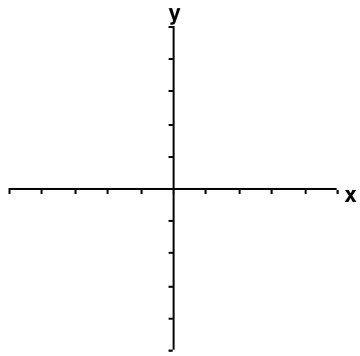
Quadratic Function



Cubic Function



Rational Function



II. Vertical and Horizontal Shifts (Pages 42–43)

> ***What you should learn***
> How to use vertical and horizontal shifts and reflections to graph functions

Let c be a positive real number. Complete the following representations of shifts in the graph of $y = f(x)$:

1) Vertical shift c units upward: ___________________________
2) Vertical shift c units downward: _________________________
3) Horizontal shift c units to the right: ______________________
4) Horizontal shift c units to the left: ________________________

Example 1: Let $f(x) = |x|$. Write the equation for the function resulting from a vertical shift of 3 units downward and a horizontal shift of 2 units to the right of the graph of $f(x)$.

III. Reflecting Graphs (Pages 44–45)

A **reflection** in the x-axis is a type of transformation of the graph of $y = f(x)$ represented by $h(x)$ = ______________. A **reflection** in the y-axis is a type of transformation of the graph of $y = f(x)$ represented by $h(x)$ = ______________.

Example 2: Let $f(x) = |x|$. Describe the graph of $g(x) = -|x|$ in terms of f.

IV. Nonrigid Transformations (Page 46)

> ***What you should learn***
> How to use nonrigid transformations to graph functions

Name three types of rigid transformations:

1)

2)

3)

Rigid transformations change only the ______________ of the graph in the coordinate plane.

Name four types of nonrigid transformations:

1)

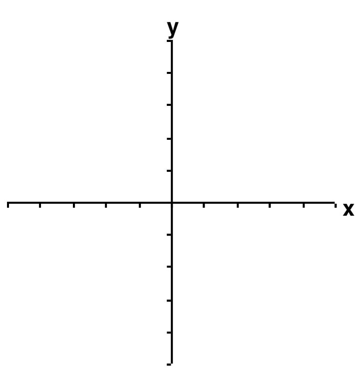
2)

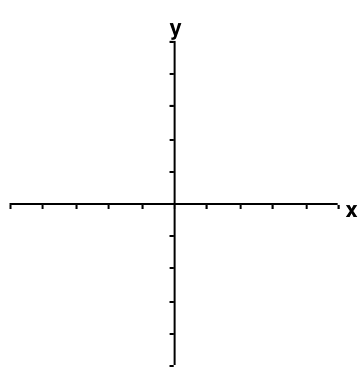
3)

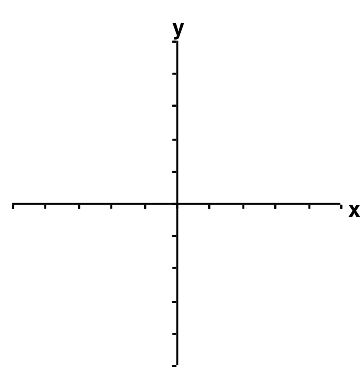
4)

A nonrigid transformation $y = cf(x)$ of the graph of $y = f(x)$ is a ______________________ if $c > 1$ or a _________________ if $0 < c < 1$. A nonrigid transformation $y = f(cx)$ of the graph of $y = f(x)$ is a ____________________________ if $c > 1$ or a ______________________________ if $0 < c < 1$.

Additional notes







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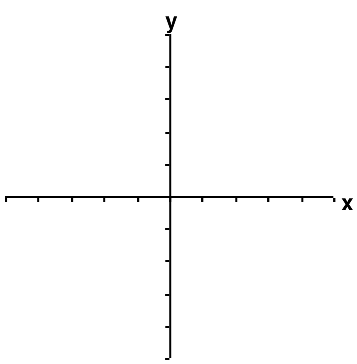

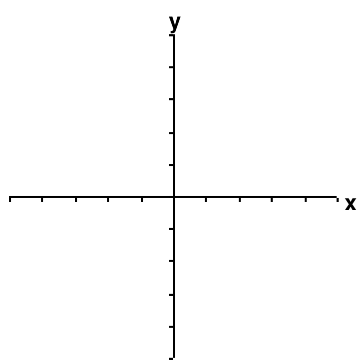

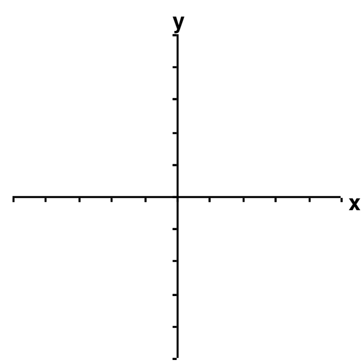

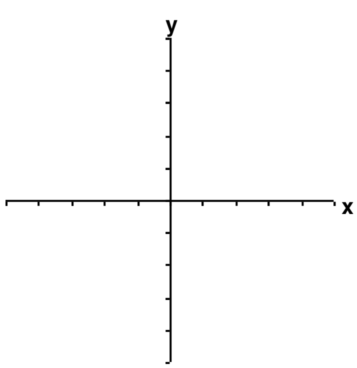

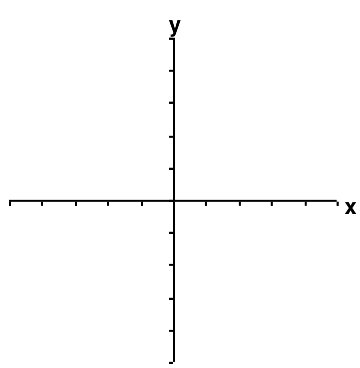

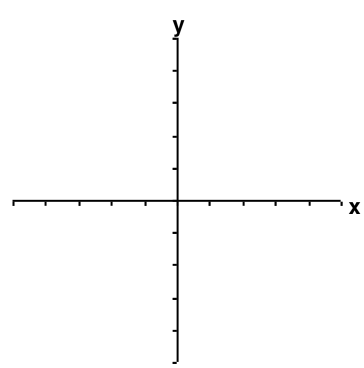

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Section 1.5 Combinations of Functions

Objective: In this lesson you learned how to find arithmetic combinations and compositions of functions.

I. Arithmetic Combinations of Functions (Pages 50–51)

> ***What you should learn***
> How to add, subtract, multiply, and divide functions

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions *f* and *g* can be combined to create new functions such as the ______________________________ ________________ of *f* and *g* to create new functions.

The domain of an arithmetic combination of functions *f* and *g* consists of ____
____
____.

Let *f* and *g* be two functions with overlapping domains. Complete the following arithmetic combinations of *f* and *g* for all *x* common to both domains:

1) Sum: $(f+g)(x) =$ ____________________________

2) Difference: $(f-g)(x) =$ ________________________

3) Product: $(fg)(x) =$ ____________________________

4) Quotient: $\left(\dfrac{f}{g}\right)(x) =$ ____________________________

To use a graphing utility to graph the sum of two functions,
____
____

Example 1: Let $f(x) = 7x - 5$ and $g(x) = 3 - 2x$. Find $(f-g)(4)$.

II. Compositions of Functions (Pages 52–54)

The **composition** of the function *f* with the function *g* is

$(f \circ g)(x) =$ ______________________.

For the composition of the function *f* with *g*, the domain of

$f \circ g$ is ____

____.

Example 2: Let $f(x) = 3x + 4$ and let $g(x) = 2x^2 - 1$. Find (a) $(f \circ g)(x)$ and (b) $(g \circ f)(x)$.

> ***What you should learn***
> How to find compositions of one function with another function

III. Applications of Combinations of Functions (Page 55)

The function $f(x) = 0.06x$ represents the sales tax owed on a purchase with a price tag of *x* dollars and the function $g(x) = 0.75x$ represents the sale price of an item with a price tag of *x* dollars during a 25% off sale. Using one of the combinations of functions discussed in this section, write the function that represents the sales tax owed on an item with a price tag of *x* dollars during a 25% off sale.

> ***What you should learn***
> How to use combinations of functions to model and solve real-life problems

Additional notes

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Section 1.6 Inverse Functions

Objective: In this lesson you learned how to find inverse functions graphically and algebraically.

Important Vocabulary	Define each term or concept.
Inverse function	
One-to-one	
Horizontal Line Test	

I. Inverse Functions (Pages 60–62)

> ***What you should learn***
> How to find inverse functions informally and verify that two functions are inverse functions of each other

For a function *f* that is defined by a set of ordered pairs, to form the inverse function of *f*, ________________________________ ____.

For a function *f* and its inverse f^{-1}, the domain of *f* is equal to ______________________, and the range of *f* is equal to ______________________.

To verify that two functions, *f* and *g*, are inverses of each other, ____ ____ ____.

Example 1: Verify that the functions $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$ are inverses of each other.

II. The Graph of an Inverse Function (Page 63)

> ***What you should learn***
> How to use graphs of functions to decide whether functions have inverse functions

If the point (*a*, *b*) lies on the graph of *f*, then the point (____________) lies on the graph of f^{-1} and vice versa. The graph of f^{-1} is a reflection of the graph of *f* in the line ______________.

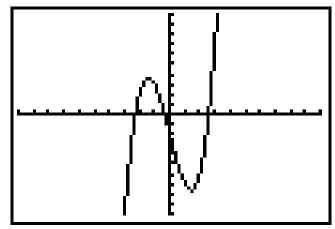
III. The Existence of an Inverse Function (Page 64)

What you should learn
How to determine whether functions are one-to-one

If a function is **one-to-one,** that means ____________________
____
____.

A function f has an inverse f^{-1} if and only if ________________
____________________.

To tell whether a function is one-to-one from its graph, ______
____
____



Example 2: Does the graph of the function at the right have an inverse function? Explain.

IV. Finding Inverse Functions Algebraically (Pages 65–66)

What you should learn
How to find inverse functions algebraically

To find the inverse of a function f algebraically,

1)

2)

3)

4)

5)

Example 3: Find the inverse (if it exists) of $f(x) = 4x - 5$.

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Section 1.7 Linear Models and Scatter Plots

Objective: In this lesson you learned how to use scatter plots and a graphing utility to find linear models for data.

Important Vocabulary	Define each term or concept.
Fitting a line to data	

I. Scatter Plots and Correlation (Pages 71–72)

What you should learn
How to construct scatter plots and interpret correlation

Many real-life situations involve finding relationships between two variables. If data are collected and written as a set of ordered pairs, the graph of such a set is called a ____________________.

For a collection of ordered pairs of the form (x, y), if y tends to increase as x increases, then the collection is said to have a(n) ________________________________. If y tends to decrease as x increases, then the collection is said to have a(n) ___________ ____________________.

II. Fitting a Line to Data (Pages 73–75)

What you should learn
How to use scatter plots and a graphing utility to find linear models for data

Describe how to fit a line to data represented in a scatter plot.

To measure how well a linear model fits the data used to find the model, ____ _______________________________.

The correlation coefficient r of a set of data varies between

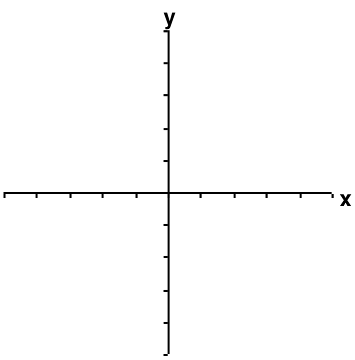
________ and ________. The closer $|r|$ is to 1, the better ______

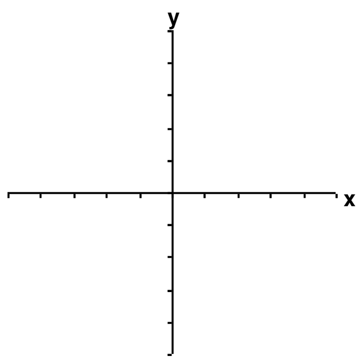
____.

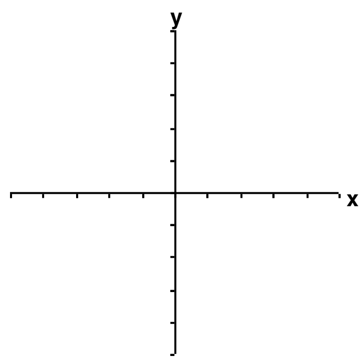
Example 1: The numbers of U.S. Navy personnel p in thousands on active duty for the years 2002 through 2006 are shown in the table. Use the regression capabilities of a graphing utility to find a linear model for the data. Let t represent the year with $t = 2$ corresponding to 2002.

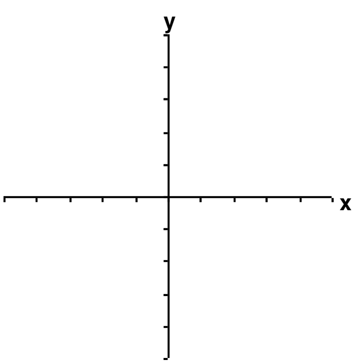
Year	2002	2003	2004	2005	2006
p	383	381	376	364	353

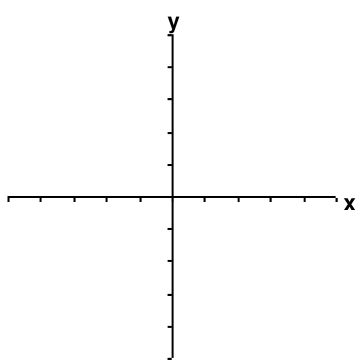
(Source: U.S. Department of Defense)

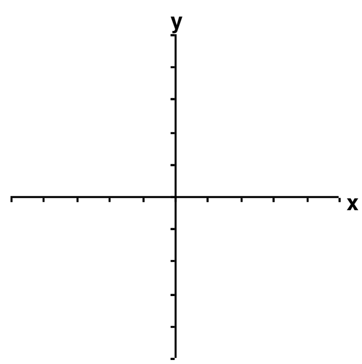












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Chapter 2 Polynomial and Rational Functions

Section 2.1 Quadratic Functions

Objective: In this lesson you learned how to sketch and analyze graphs of quadratic functions.

Important Vocabulary	Define each term or concept.
Constant function	
Linear function	
Quadratic function	
Axis of symmetry	
Vertex	

I. The Graph of a Quadratic Function (Pages 90–92)

What you should learn
How to analyze graphs of quadratic functions

Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. A **polynomial function of *x* with degree *n*** is

____

A quadratic function is a polynomial function of _____________ degree. The graph of a quadratic function is a special "U"-shaped curve called a(n) ___________________.

If the leading coefficient of a quadratic function is positive, the graph of the function opens _____________ and the vertex of the parabola is the _______________ point on the graph. If the leading coefficient of a quadratic function is negative, the graph of the function opens ________________ and the vertex of the parabola is the _______________ point on the graph.

II. The Standard Form of a Quadratic Function
(Pages 93–94)

> ***What you should learn***
> How to write quadratic functions in standard form and use the results to sketch graphs of functions

The **standard form of a quadratic function** is

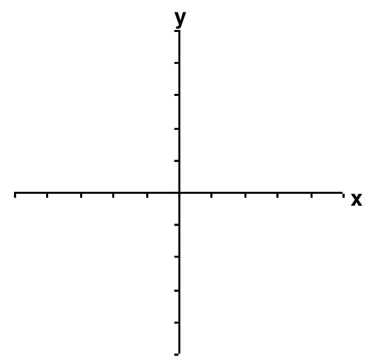
____ .

For a quadratic function in standard form, the axis of the associated parabola is ___________ and the vertex is _______________.

To write a quadratic function in standard form, __________ ____.

To find the x-intercepts of the graph of $f(x) = ax^2 + bx + c$, ____.

Example 1: Sketch the graph of $f(x) = x^2 + 2x - 8$ and identify the vertex, axis, and x-intercepts of the parabola.



III. Finding Minimum and Maximum Values (Page 95)

> ***What you should learn***
> How to find minimum and maximum values of quadratic functions in real-life applications

For a quadratic function in the form $f(x) = ax^2 + bx + c$, when $a > 0$, f has a minimum that occurs at ____________________.
When $a < 0$, f has a maximum that occurs at ________________.
To find the minimum or maximum value, ___________________ ____.

Example 2: Find the minimum value of the function $f(x) = 3x^2 - 11x + 16$. At what value of x does this minimum occur?

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Section 2.2 Polynomial Functions of Higher Degree

Objective: In this lesson you learned how to sketch and analyze graphs of polynomial functions.

Important Vocabulary Define each term or concept.

Continuous

Extrema

Relative minimum

Relative maximum

Repeated zero

Multiplicity

Intermediate Value Theorem

I. Graphs of Polynomial Functions (Pages 100–101)

What you should learn
How to use transformations to sketch graphs of polynomial functions

Name two basic features of the graphs of polynomial functions.

1)
2)

Will the graph of $g(x) = x^7$ look more like the graph of $f(x) = x^2$ or the graph of $f(x) = x^3$? Explain.

II. The Leading Coefficient Test (Pages 102–103)

What you should learn
How to use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions

State the **Leading Coefficient Test.**

1.
   a.
   b.
2.
   a.
   b.

Example 1: Describe the left and right behavior of the graph of $f(x) = 1 - 3x^2 - 4x^6$.

III. Zeros of Polynomial Functions (Pages 104–107)

What you should learn
How to find and use zeros of polynomial functions as sketching aids

Let *f* be a polynomial function of degree *n*. The function *f* has at most ___________ real zeros. The graph of *f* has at most ____________ relative extrema.

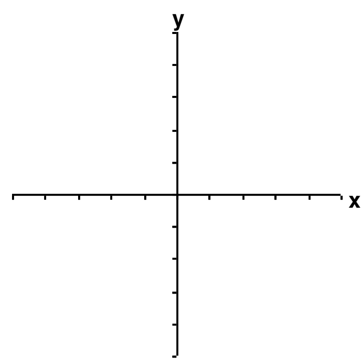
Let *f* be a polynomial function and let *a* be a real number. List four equivalent statements about the real zeros of *f*.
1)

2)

3)

4)

If a polynomial function *f* has a repeated zero $x = 3$ with multiplicity 4, the graph of *f* ______________ the *x*-axis at $x =$ ______. If *f* has a repeated zero $x = 4$ with multiplicity 3, the graph of *f* ______________ the *x*-axis at $x =$ _______.



Example 2: Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + 3$.

IV. The Intermediate Value Theorem (Page 108)

What you should learn
How to use the Intermediate Value Theorem to help locate zeros of polynomial functions

Interpret the meaning of the Intermediate Value Theorem.

Describe how the Intermediate Value Theorem can help in locating the real zeros of a polynomial function *f*.

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Section 2.3 Real Zeros of Polynomial Functions

Objective: In this lesson you learned how to use long division and synthetic division to divide polynomials by other polynomials and how to find the rational and real zeros of polynomial functions.

Important Vocabulary Define each term or concept.

Long division of polynomials

Division Algorithm

Synthetic division

Remainder Theorem

Factor Theorem

Upper bound

Lower bound

I. Long Division of Polynomials (Pages 113–115)

What you should learn
How to use long division to divide polynomials by other polynomials

When dividing a polynomial $f(x)$ by another polynomial $d(x)$, if the remainder $r(x) = 0$, $d(x)$ ______________________ into $f(x)$.

The rational expression $f(x)/d(x)$ is **improper** if ______________
____.

The rational expression $r(x)/d(x)$ is **proper** if _______________
____.

Before applying the Division Algorithm, you should __________
____
____
____.

Example 1: Divide $3x^3 + 4x - 2$ by $x^2 + 2x + 1$.

II. Synthetic Division (Page 116)

> ***What you should learn***
> How to use synthetic division to divide polynomials by binomials of the form $(x - k)$

Can synthetic division be used to divide a polynomial by $x^2 - 5$? Explain.

Can synthetic division be used to divide a polynomial by $x + 4$? Explain.

Example 2: Fill in the following synthetic division array to divide $2x^4 + 5x^2 - 3$ by $x - 5$. Then carry out the synthetic division and indicate which entry represents the remainder.

III. The Remainder and Factor Theorems (Pages 117–118)

> ***What you should learn***
> How to use the Remainder and Factor Theorems

To use the Remainder Theorem to evaluate a polynomial function $f(x)$ at $x = k$, ________________________________

____.

Example 3: Use the Remainder Theorem to evaluate the function $f(x) = 2x^4 + 5x^2 - 3$ at $x = 5$.

To use the Factor Theorem to show that $(x - k)$ is a factor of a polynomial function $f(x)$, ________________________________

____

____

____.

List three facts about the remainder r, obtained in the synthetic division of $f(x)$ by $x - k$:

1)

2)

3)

IV. The Rational Zero Test (Pages 119–120)

What you should learn
How to use the Rational Zero Test to determine possible rational zeros of polynomial functions

Describe the purpose of the Rational Zero Test.

State the **Rational Zero Test.**

Describe how to use the Rational Zero Test.

Example 4: List the possible rational zeros of the polynomial function $f(x) = 3x^5 + x^4 + 4x^3 - 2x^2 + 8x - 5$.

List some strategies that can be used to shorten the search for actual zeros among a list of possible rational zeros.

V. Other Tests for Zeros of Polynomials (Pages 121–123)

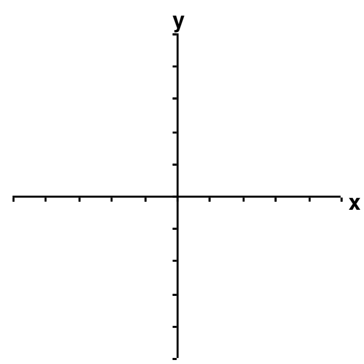
> ***What you should learn***
> How to use Descartes's Rule of Signs and the Upper and Lower Bound Rules to find zeros of polynomials

State the Upper and Lower Bound Rules.

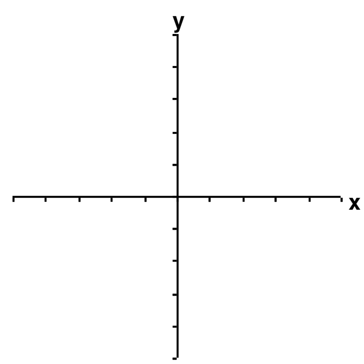
1.

2.

Explain how the Upper and Lower Bound Rules can be useful in the search for the real zeros of a polynomial function.



Additional notes



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Section 2.4 Complex Numbers

Objective: In this lesson you learned how to perform operations with complex numbers.

Important Vocabulary	Define each term or concept.
Complex number	
Complex conjugates	

I. The Imaginary Unit *i* (Page 128)

What you should learn
How to use the imaginary unit i to write complex numbers

Mathematicians created an expanded system of numbers using the **imaginary unit *i*,** defined as $i =$ ___________, because

____

By definition, $i^2 =$ _____________.

For the complex number $a + bi$, if $b = 0$, the number $a + bi = a$ is a(n) ___________________. If $b \neq 0$, the number $a + bi$ is a(n) ___________________________. If $a = 0$, the number $a + bi = b$, where $b \neq 0$, is called a(n) ___________________________.

The set of complex numbers consists of the set of _____________________ and the set of ___________________________.

Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other if ___________________________ .

II. Operations with Complex Numbers (Pages 129–130)

What you should learn
How to add, subtract, and multiply complex numbers

To add two complex numbers, ___________________________

____.

To subtract two complex numbers, ________________________

____.

The additive identity in the complex number system is ___________.

The additive inverse of the complex number $a + bi$ is

______________________.

Example 1: Perform the operations:
$(5 - 6i) - (3 - 2i) + 4i$

To multiply two complex numbers $a + bi$ and $c + di$, ________

____

____

____.

Example 2: Multiply: $(5 - 6i)(3 - 2i)$

III. Complex Conjugates (Page 131)

What you should learn
How to use complex conjugates to write the quotient of two complex numbers in standard form

The product of a pair of complex conjugates is a(n) _____________________ number.

To find the quotient of the complex numbers $a + bi$ and $c + di$, where c and d are not both zero, _________________________

____.

Example 3: Divide $(1 + i)$ by $(2 - i)$. Write the result in standard form.

IV. Complex Solutions of Quadratic Equations (Page 132)

What you should learn
How to find complex solutions of quadratic equations

When using the Quadratic Formula to solve a quadratic equation, you may obtain a result such as $\sqrt{-7}$, which is not a___________ _______________. By factoring out $i = \sqrt{-1}$, you can write this number in __________________________________.

If a is a positive number, then the **principal square root** of the negative number $-a$ is defined as _______________.

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Section 2.5 The Fundamental Theorem of Algebra

Objective: In this lesson you learned how to determine the numbers of zeros of polynomial functions and find them.

Important Vocabulary	Define each term or concept.
Fundamental Theorem of Algebra	
Linear Factorization Theorem	
Conjugates	

I. The Fundamental Theorem of Algebra (Page 135)

What you should learn
How to use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function

In the complex number system, every *n*th-degree polynomial function has _________________ zeros.

Example 1: How many zeros does the polynomial function $f(x) = 5 - 2x^2 + x^3 - 12x^5$ have?

An *n*th-degree polynomial can be factored into
___________________ linear factors.

II. Finding Zeros of a Polynomial Function (Page 136)

What you should learn
How to find all zeros of polynomial functions, including complex zeros

Remember that the *n* zeros of a polynomial function can be real or complex, and they may be _______________________.

Example 2: List all of the zeros of the polynomial function $f(x) = x^3 - 2x^2 + 36x - 72$.

III. Conjugate Pairs (Page 137)

What you should learn
How to find conjugate pairs of complex zeros

Let $f(x)$ be a polynomial function that has real coefficients. If $a + bi$, where $b \neq 0$, is a zero of the function, then we know that ________________ is also a zero of the function.

IV. Factoring a Polynomial (Pages 138–139)

What you should learn
How to find zeros of polynomials by factoring

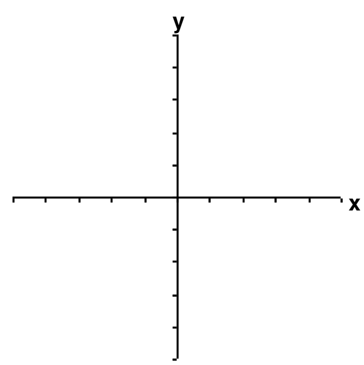
To write a polynomial of degree $n > 0$ with real coefficients as a product without complex factors, write the polynomial as ______
____
____.

A quadratic factor with no real zeros is said to be
____.

Example 3: Write the polynomial $f(x) = x^4 + 5x^2 - 36$
(a) as the product of linear factors and quadratic factors that are irreducible over the reals, and
(b) in completely factored form.

Explain why a graph cannot be used to locate complex zeros.

Additional notes



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Section 2.6 Rational Functions and Asymptotes

Objective: In this lesson you learned how to determine the domains and find asymptotes of rational functions.

Important Vocabulary	Define each term or concept.
Rational function	
Vertical asymptote	
Horizontal asymptote	

I. Introduction to Rational Functions (Page 142)

What you should learn
How to find the domains of rational functions

The domain of a rational function of x includes all real numbers except ____.

To find the domain of a rational function of x, _______________
____
____
____.

Example 1: Find the domain of the function $f(x) = \dfrac{1}{x^2 - 9}$.

II. Vertical and Horizontal Asymptotes (Pages 143–145)

What you should learn
How to find vertical and horizontal asymptotes of graphs of rational functions

The notation "$f(x) \to 5$ as $x \to \infty$" means ________________
____.

Let f be the rational function given by

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors.

1) The graph of f has vertical asymptotes at _____________
_____________________.

2) The graph of f has at most one horizontal asymptote determined by ________________________________ __________.

   a) If $n < m$, ______________________________ ______________________________.

   b) If $n = m$, ______________________________ ____________________ .

   c) If $n > m$, the graph of f has ________________ ________________.

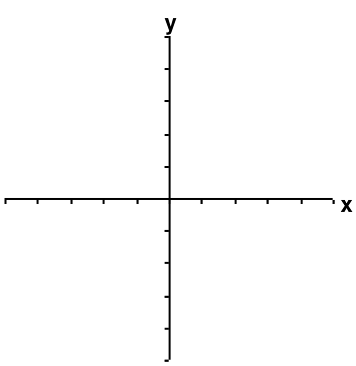
Example 2: Find the asymptotes of the function

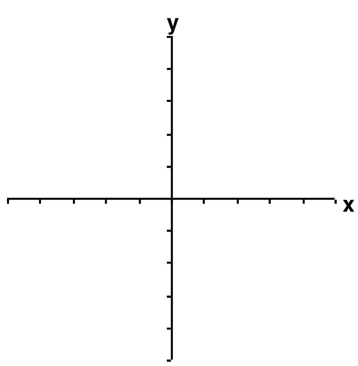
$$f(x) = \frac{2x-1}{x^2-x-6}.$$

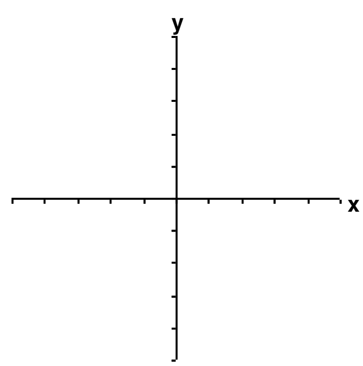
III. Application of Rational Functions (Page 146)

> ***What you should learn***
> How to use rational functions to model and solve real-life problems

Give an example of asymptotic behavior that occurs in real life.







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Section 2.7 Graphs of Rational Functions

Objective: In this lesson you learned how to sketch graphs of rational functions.

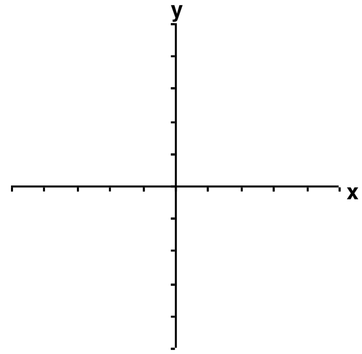
Important Vocabulary	Define each term or concept.
Slant (or oblique) asymptote	

I. The Graph of a Rational Function (Pages 151–154)

What you should learn
How to analyze and sketch graphs of rational functions

List the guidelines for sketching the graph of the rational function $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials.

Example 1: Sketch the graph of $f(x) = \dfrac{3x}{x+4}$.



II. Slant Asymptotes (Page 155)

> ***What you should learn***
> How to sketch graphs of rational functions that have slant asymptotes

Describe how to find the equation of a slant asymptote.

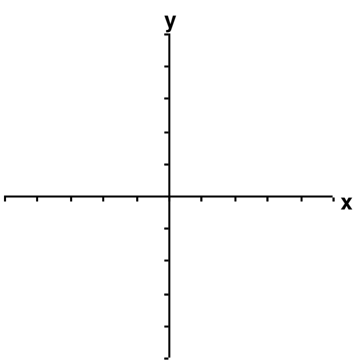
Example 2: Decide whether each of the following rational functions has a slant asymptote. If so, find the equation of the slant asymptote.

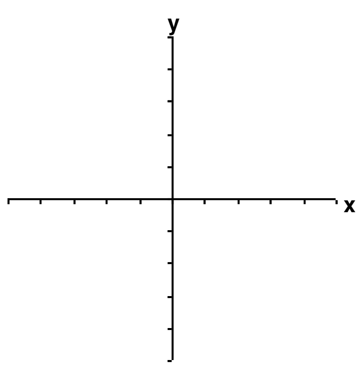
(a) $f(x) = \dfrac{x^3 - 1}{x^2 + 3x + 5}$ (b) $f(x) = \dfrac{3x^3 + 2}{2x - 5}$

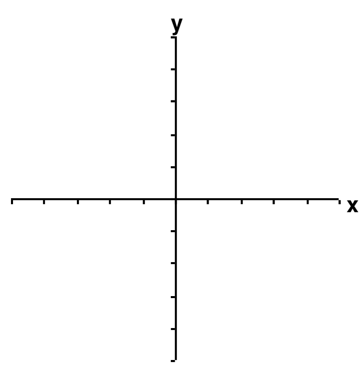
III. Applications of Graphs of Rational Functions (Page 156)

> ***What you should learn***
> How to use graphs of rational functions to model and solve real-life problems

Describe a real-life situation in which a graph of a rational function would be helpful when solving a problem.







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Section 2.8 Quadratic Models

Objective: In this lesson you learned how to classify scatter plots and use a graphing utility to find quadratic models for data.

I. Classifying Scatter Plots (Page 161)

What you should learn
How to classify scatter plots

Describe how to decide whether a set of data can be modeled by a linear model.

Describe how to decide whether a set of data can be modeled by a quadratic model.

II. Fitting a Quadratic Model to Data (Pages 162–163)

What you should learn
How to use scatter plots and a graphing utility to find quadratic models for data

Once it has been determined that a quadratic model is appropriate for a set of data, a quadratic model can be fit to data by ____

____.

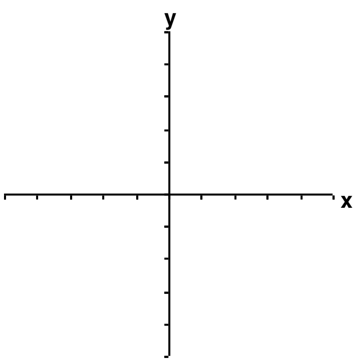
Example 1: Find a model that best fits the data given in the table.

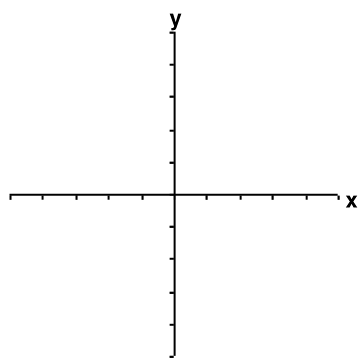
x	–1	0	2	5	9	12	15
y	8.7	3.45	–5.55	–15.3	–21.3	–20.55	–15.3

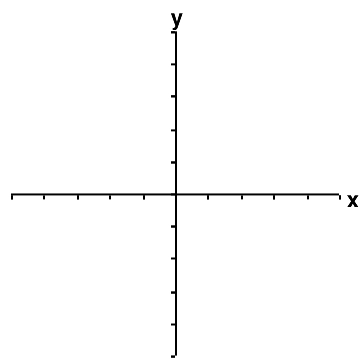
III. Choosing a Model (Page 164)

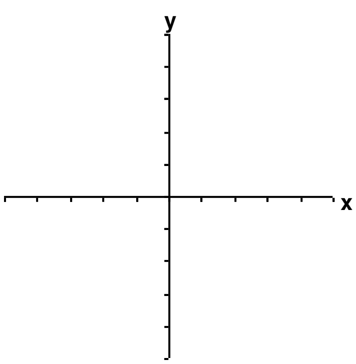
> ***What you should learn***
> How to choose a model that best fits a set of data

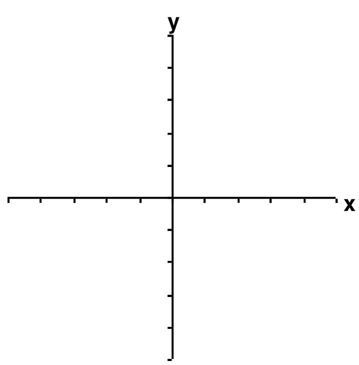
If it isn't easy to tell from a scatter plot which type of model a set of data would best be modeled by, you should ____________ ____ ____ ____.

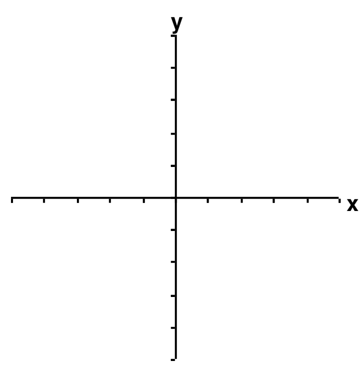












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Chapter 3 Exponential and Logarithmic Functions

Section 3.1 Exponential Functions and Their Graphs

Objective: In this lesson you learned how to recognize, evaluate, and graph exponential functions.

Important Vocabulary	Define each term or concept.
Transcendental functions	
Natural base *e*	

I. Exponential Functions (Page 180)

> ***What you should learn***
> How to recognize and evaluate exponential functions with base a

Polynomial functions and rational functions are examples of

___________________ functions.

The **exponential function *f* with base *a*** is denoted by

__________________, where $a > 0$, $a \neq 1$, and x is any real number.

Example 1: Use a calculator to evaluate the expression $5^{3/5}$.

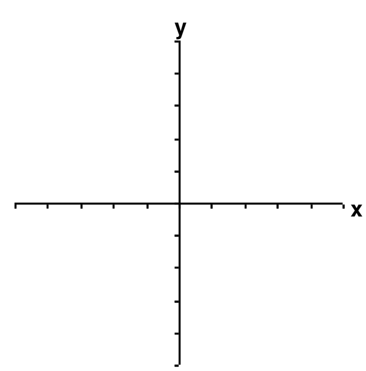
II. Graphs of Exponential Functions (Pages 181–183)

> ***What you should learn***
> How to graph exponential functions with base a

For $a > 1$, is the graph of $f(x) = a^x$ increasing or decreasing over its domain? __________________________

For $a > 1$, is the graph of $g(x) = a^{-x}$ increasing or decreasing over its domain? __________________________

For the graph of $y = a^x$ or $y = a^{-x}$, $a > 1$, the domain is ____________________, the range is ________________, and the intercept is ________________. Also, both graphs have ________________________ as a horizontal asymptote.



Example 2: Sketch the graph of the function $f(x) = 3^{-x}$.

III. The Natural Base *e* (Pages 184–185)

What you should learn
How to recognize, evaluate, and graph exponential functions with base *e*

The **natural exponential function** is given by the function ____________________.

Example 3: Use a calculator to evaluate the expression $e^{3/5}$.

For the graph of $f(x) = e^x$, the domain is __________________, the range is _______________, and the intercept is ____________.

The number *e* can be approximated by the expression __________________ for large values of *x*.

IV. Applications (Pages 186–188)

What you should learn
How to use exponential functions to model and solve real-life problems

After *t* years, the balance *A* in an account with principal *P* and annual interest rate *r* (in decimal form) is given by the formulas:

For *n* compoundings per year: _______________________

For continuous compounding: ________________

Example 4: Find the amount in an account after 10 years if \$6000 is invested at an interest rate of 7%,
(a) compounded monthly.
(b) compounded continuously.

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Section 3.2 Logarithmic Functions and Their Graphs

Objective: In this lesson you learned how to recognize, evaluate, and graph logarithmic functions.

Important Vocabulary	Define each term or concept.
Common logarithmic function	
Natural logarithmic function	

What you should learn
How to recognize and evaluate logarithmic functions with base *a*

I. Logarithmic Functions (Pages 192–193)

The logarithmic function with base *a* is the ________________ ___________________ of the exponential function $f(x) = a^x$.

The **logarithmic function with base *a*** is defined as _____________________ , for $x > 0$, $a > 0$, and $a \neq 1$, if and only if $x = a^y$. The notation "$\log_a x$" is read as "_____________ ________________________."

The equation $x = a^y$ in exponential form is equivalent to the equation ____________________ in logarithmic form.

When evaluating logarithms, remember that a logarithm is a(n) ______________. This means that $\log_a x$ is the ______________ to which *a* must be raised to obtain ________.

Example 1: Use the definition of logarithmic function to evaluate $\log_5 125$.

Example 2: Use a calculator to evaluate $\log_{10} 300$.

Complete the following properties of logarithms:

1) $\log_a 1 =$ ___________ 2) $\log_a a =$ ___________

3) $\log_a a^x =$ ___________ and $a^{\log_a x} =$ ___________

4) If $\log_a x = \log_a y$, then ______________.

Example 3: Solve the equation $\log_7 x = 1$ for x.

II. Graphs of Logarithmic Functions (Pages 194–195)

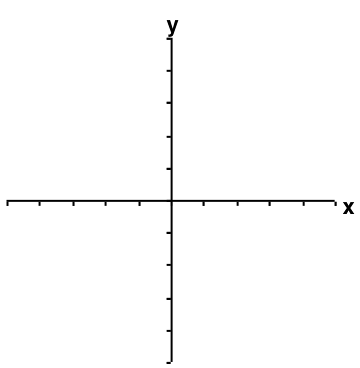
What you should learn
How to graph logarithmic functions with base a

For $a > 1$, is the graph of $f(x) = \log_a x$ increasing or decreasing over its domain? __________________________

For the graph of $f(x) = \log_a x$, $a > 1$, the domain is ___________________, the range is _________________, and the intercept is _______________.

Also, the graph has ________________________ as a vertical asymptote. The graph of $f(x) = \log_a x$ is a reflection of the graph of $f(x) = a^x$ in _____________________.

Example 4: Sketch the graph of the function $f(x) = \log_3 x$.



III. The Natural Logarithmic Function (Pages 196–197)

> ***What you should learn***
> How to recognize, evaluate, and graph natural logarithmic functions

Complete the following properties of natural logarithms:

1) $\ln 1 =$ ____________ 2) $\ln e =$ ____________

3) $\ln e^x =$ ____________ and $e^{\ln x} =$ ____________

4) If $\ln x = \ln y$, then ______________.

Example 5: Use a calculator to evaluate $\ln 10$.

Example 6: Find the domain of the function $f(x) = \ln(x+3)$.

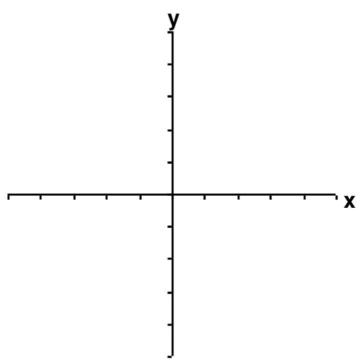
IV. Applications of Logarithmic Functions (Page 198)

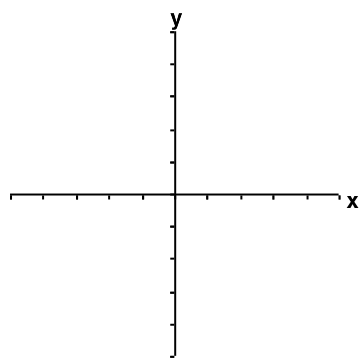
> ***What you should learn***
> How to use logarithmic functions to model and solve real-life problems

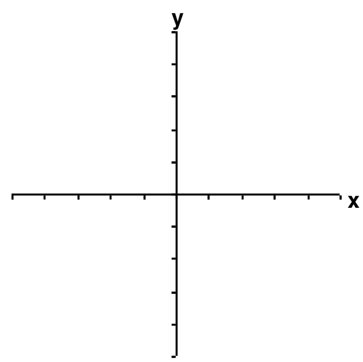
Describe a real-life situation in which logarithms are used.

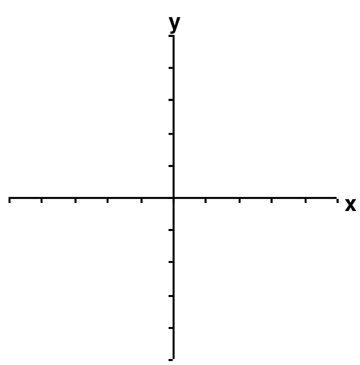
Example 7: A principal P, invested at 6% interest and compounded continuously, increases to an amount K times the original principal after t years, where t is given by $t = \dfrac{\ln K}{0.06}$. How long will it take the original investment to double in value? To triple in value?

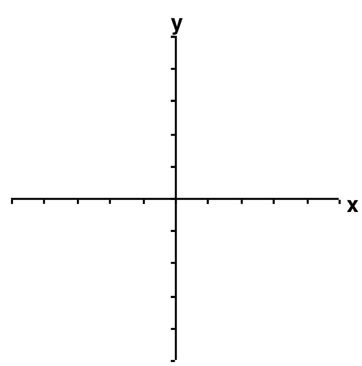
Additional notes

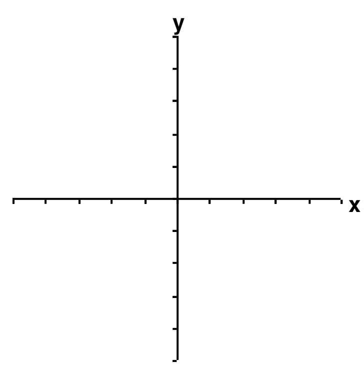












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Section 3.3 Properties of Logarithms

Objective: In this lesson you learned how to rewrite logarithmic functions with different bases and how to use properties of logarithms to evaluate, rewrite, expand, or condense logarithmic expressions.

I. Change of Base (Page 203)

What you should learn
How to rewrite logarithms with different bases

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. The **change-of-base formula** states that:

Explain how to use a calculator to evaluate $\log_8 20$.

II. Properties of Logarithms (Page 204)

What you should learn
How to use properties of logarithms to evaluate or rewrite logarithmic expressions

Let a be a positive number such that $a \neq 1$; let n be a real number; and let u and v be positive real numbers. Complete the following properties of logarithms:

1. $\log_a(uv)$ = ______________________

2. $\log_a \frac{u}{v}$ = ______________________

3. $\log_a u^n$ = ______________________

III. Rewriting Logarithmic Expressions (Page 205)

What you should learn
How to use properties of logarithms to expand or condense logarithmic expressions

To expand a logarithmic expression means to _______________
____
____
____.

Example 1: Expand the logarithmic expression $\ln \frac{xy^4}{2}$.

To condense a logarithmic expression means to __________

____

____.

Example 2: Condense the logarithmic expression $3\log x + 4\log(x-1)$.

IV. Applications of Properties of Logarithms (Page 206)

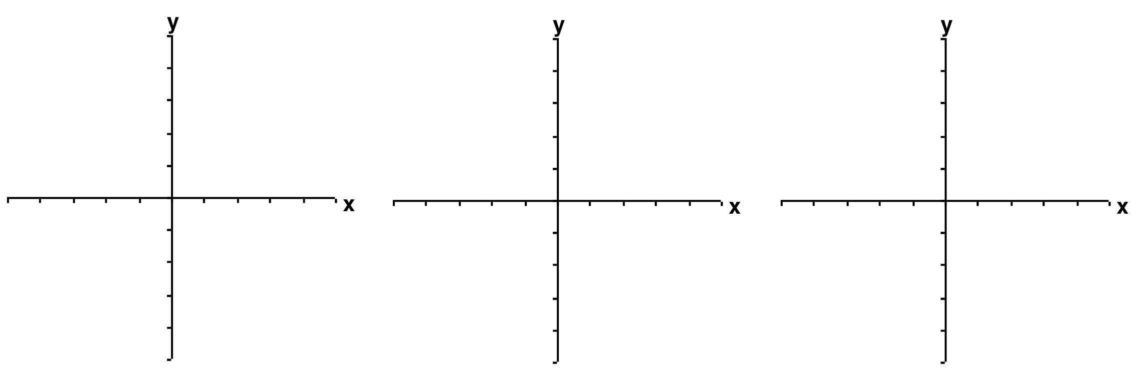
What you should learn
How to use logarithmic functions to model and solve real-life problems

One way of finding a model for a set of nonlinear data is to take the natural log of each of the *x*-values and *y*-values of the data set. If the points are graphed and fall on a straight line, then the *x*-values and the *y*-values are related by the equation:

_________________________, where *m* is the slope of the straight line.

Example 3: Find a natural logarithmic equation for the following data that expresses *y* as a function of *x*.

x	2.718	7.389	20.086	54.598
y	7.389	54.598	403.429	2980.958



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Section 3.4 Solving Exponential and Logarithmic Equations

Objective: In this lesson you learned how to solve exponential and logarithmic equations.

I. Introduction (Page 210)

> ***What you should learn***
> How to solve simple exponential and logarithmic equations

State the One-to-One Property for exponential equations.

State the One-to-One Property for logarithmic equations.

State the Inverse Properties for exponential equations and for logarithmic equations.

Describe some strategies for using the One-to-One Properties and the Inverse Properties to solve exponential and logarithmic equations.

-
-
-

Example 1: (a) Solve $\log_8 x = \frac{1}{3}$ for x.

(b) Solve $5^x = 0.04$ for x.

II. Solving Exponential Equations (Pages 211–212)

> ***What you should learn***
> How to solve more complicated exponential equations

Describe how to solve the exponential equation $10^x = 90$ algebraically.

Example 2: Solve $e^{x-2} - 7 = 59$ for x. Round to three decimal places.

III. Solving Logarithmic Equations (Pages 213–215)

What you should learn
How to solve more complicated logarithmic equations

Describe how to solve the logarithmic equation $\log_6(4x - 7) = \log_6(8 - x)$ algebraically.

Example 3: Solve $4 \ln 5x = 28$ for x. Round to three decimal places.

Describe a method that can be used to approximate the solutions of an exponential or logarithmic equation using a graphing utility.

IV. Applications of Solving Exponential and Logarithmic Equations (Page 216)

What you should learn
How to use exponential and logarithmic equations to model and solve real-life problems

Example 4: Use the formula for continuous compounding, $A = Pe^{rt}$, to find how long it will take \$1500 to triple in value if it is invested at 12% interest, compounded continuously.

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Section 3.5 Exponential and Logarithmic Models

Objective: In this lesson you learned how to use exponential growth models, exponential decay models, Gaussian models, logistic models, and logarithmic models to solve real-life problems.

Important Vocabulary Define each term or concept.

Bell-shaped curve

Logistic curve

Sigmoidal curve

I. Introduction (Page 221)

What you should learn
How to recognize the five most common types of models involving exponential or logarithmic functions

The **exponential growth model** is ____________________.

The **exponential decay model** is ____________________.

The **Gaussian model** is ______________________ .

The **logistic growth model** is ________________________ .

Logarithmic models are _________________ and

______________________.

II. Exponential Growth and Decay (Pages 222–224)

What you should learn
How to use exponential growth and decay functions to model and solve real-life problems

Example 1: Suppose a population is growing according to the model $P = 800e^{0.05t}$, where t is given in years.
(a) What is the initial size of the population?
(b) How long will it take this population to double?

To estimate the age of dead organic matter, scientists use the carbon dating model ______________________________, which denotes the ratio R of carbon 14 to carbon 12 present at any time t (in years).

Example 2: The ratio of carbon 14 to carbon 12 in a fossil is $R = 10^{-16}$. Find the age of the fossil.

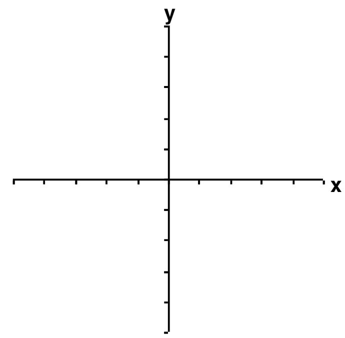
III. Gaussian Models (Page 225)

> ***What you should learn***
> How to use Gaussian functions to model and solve real-life problems

The Gaussian model is commonly used in probability and statistics to represent populations that are ______________ ____________________.

On a bell-shaped curve, the average value for a population is where the ________________________ of the function occurs.

Example 3: Draw the basic form of the graph of a Gaussian model.

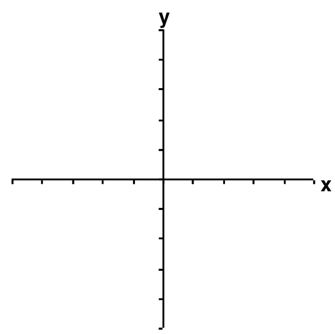


IV. Logistic Growth Models (Page 226)

> ***What you should learn***
> How to use logistic growth functions to model and solve real-life problems

Give an example of a real-life situation that is modeled by a logistic growth model.

Example 4: Draw the basic form of the graph of a logistic growth model.



V. Logarithmic Models (Page 227)

> ***What you should learn***
> How to use logarithmic functions to model and solve real-life problems

Example 5: The number of kitchen widgets y (in millions) demanded each year is given by the model $y = 2 + 3\ln(x+1)$, where $x = 0$ represents the year 2000 and $x \geq 0$. Find the year in which the number of kitchen widgets demanded will be 8.6 million.

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Section 3.6 Nonlinear Models

Objective: In this lesson you learned how to fit exponential, logarithmic, power, and logistic models to sets of data.

I. Classifying Scatter Plots (Page 233)

> ***What you should learn***
> How to classify scatter plots

When faced with a set of data to be modeled, what is a good first step in selecting which type of model will best fit the data?

II. Fitting Nonlinear Models to Data (Pages 234–235)

> ***What you should learn***
> How to use scatter plots and a graphing utility to find models for data and choose the model that best fits a set of data

Describe how to use a graphing utility to fit a nonlinear model to data.

Example 2: Find an appropriate model, either logarithmic or exponential, for the data in the following table.

x	1	3	5	7	9
y	1.120	2.195	4.303	8.433	16.529

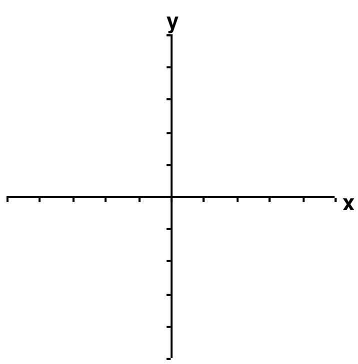
III. Modeling With Exponential and Logistic Functions
(Pages 236–237)

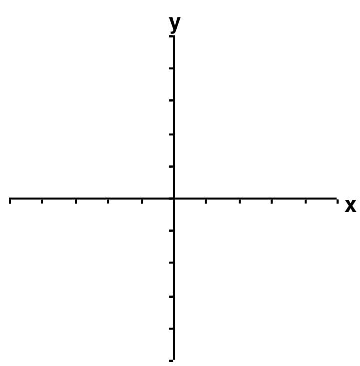
What you should learn
How to use a graphing utility to find exponential and logistic models for data

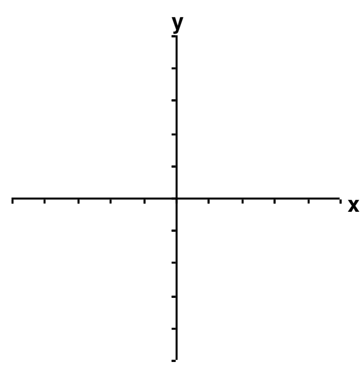
Example 3: Find a logistic model for the data in the following table.

x	0	10	15	20	25	30
y	5	27	50	73	88	95

Additional notes







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Chapter 4 Trigonometric Functions

Section 4.1 Radian and Degree Measure

Objective: In this lesson you learned how to describe an angle and to convert between degree and radian measure.

Important Vocabulary Define each term or concept.

Trigonometry

Central angle of a circle

Complementary angles

Supplementary angles

Degree

I. Angles (Page 254)

What you should learn
How to describe angles

An **angle** is determined by ______________________________

____.

The **initial side** of an angle is ____________________________

______________________________________.

The **terminal side** of an angle is __________________________

____.

The **vertex** of an angle is ________________________________

____.

An angle is in **standard position** when ____________________

____

____.

A **positive angle** is generated by a ________________________ rotation; whereas a **negative angle** is generated by a __________________________ rotation.

If two angles are **coterminal,** then they have ________________

____.

What you should learn
How to use radian measure

II. Radian Measure (Pages 255–256)

The measure of an angle is determined by ________________
____.

One **radian** is the measure of a central angle θ that __________
____.

Algebraically this means that $\theta =$ ________________
____.

A central angle of one full revolution (counterclockwise) corresponds to an arc length of $s =$ ____________.

The radian measure of an angle of one full revolution is ___________ radians. A half revolution corresponds to an angle of ___________ radians. Similarly $\frac{1}{4}$ revolution corresponds to an angle of ___________ radians, and $\frac{1}{6}$ revolution corresponds to an angle of ___________ radians.

Angles with measures between 0 and $\pi/2$ radians are ______________ angles. Angles with measures between $\pi/2$ and π radians are _______________ angles.

To find an angle that is coterminal to a given angle θ, ________
____.

Example 1: Find an angle that is coterminal with $\theta = -\pi/8$.

Example 2: Find the supplement of $\theta = \pi/4$.

III. Degree Measure (Pages 257–258)

> ***What you should learn***
> How to use degree measure and convert between degree and radian measure

A full revolution (counterclockwise) around a circle corresponds to _____________ degrees. A half revolution around a circle corresponds to _____________ degrees.

To convert degrees to radians, ____________________________

____.

To convert radians to degrees, ____________________________

____.

Example 3: Convert 120° to radians.

Example 4: Convert $9\pi/8$ radians to degrees.

Example 5: Complete the following table of equivalent degree and radian measures for common angles.

θ (degrees)	0°		45°		90°		270°
θ (radians)		$\pi/6$		$\pi/3$		π	

IV. Linear and Angular Speed (Pages 259–260)

> ***What you should learn***
> How to use angles to model and solve real-life problems

For a circle of radius r, a central angle θ intercepts an arc of length s given by _________________, where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals _________________________________.

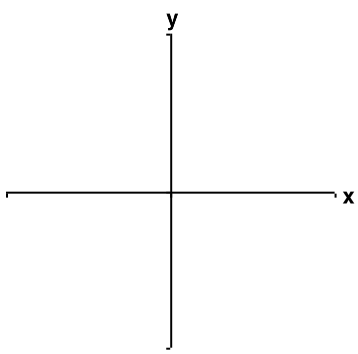
Consider a particle moving at constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** of the particle is

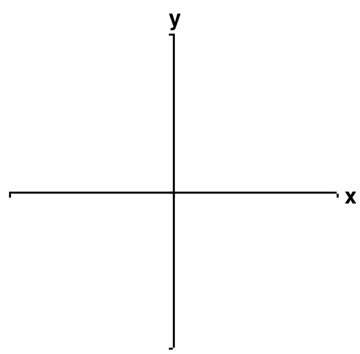
linear speed = _____________________________________

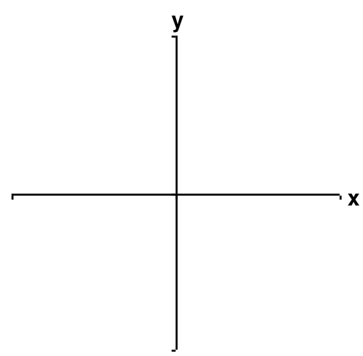
If θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** of the particle is

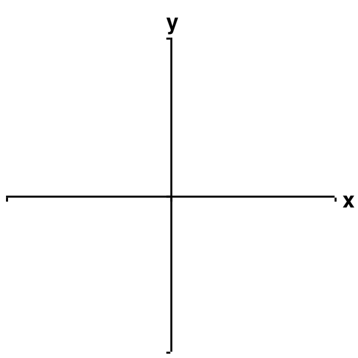
angular speed = ____________________________________

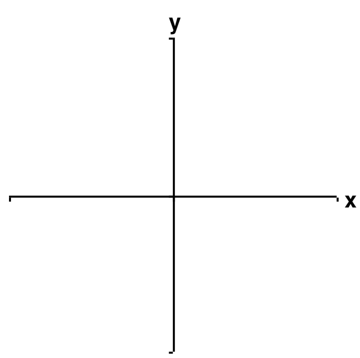
Example 6: A 6-inch-diameter gear makes 2.5 revolutions per second. Find the angular speed of the gear in radians per second.

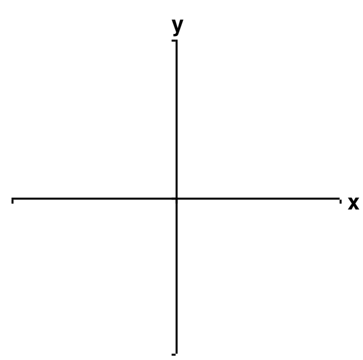












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Section 4.2 Trigonometric Functions: The Unit Circle

Objective: In this lesson you learned how to identify a unit circle and describe its relationship to real numbers.

Important Vocabulary	Define each term or concept.
Unit circle	
Periodic	
Period	

I. The Unit Circle (Page 265)

> ***What you should learn***
> How to identify a unit circle and describe its relationship to real numbers

As the real number line is wrapped around the unit circle, each real number t corresponds to ______________________________ ________________________________.

The real number 2π corresponds to the point ________________ on the unit circle.

Each real number t also corresponds to a ___________________ (in standard position) whose radian measure is t. With this interpretation of t, the arc length formula $s = r\theta$ (with $r = 1$) indicates that ____ ____.

II. The Trigonometric Functions (Pages 266–267)

> ***What you should learn***
> How to evaluate trigonometric functions using the unit circle

The coordinates x and y are two functions of the real variable t. These coordinates can be used to define six trigonometric functions of t. List the abbreviation for each trigonometric function.

Sine	_____________	**Cosecant**	_____________
Cosine	_____________	**Secant**	_____________
Tangent	_____________	**Cotangent**	_____________

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t. Complete the following definitions of the trigonometric functions:

$\sin t =$ ____________ $\cos t =$ ____________

$\tan t =$ _____________ $\cot t =$ _____________

$\sec t =$ _____________ $\csc t =$ _____________

The cosecant function is the reciprocal of the ______________ function. The cotangent function is the reciprocal of the ________________ function. The secant function is the reciprocal of the ________________ function.

Complete the following table showing the correspondence between the real number t and the point (x, y) on the unit circle when the unit circle is divided into eight equal arcs.

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x								
y								

Complete the following table showing the correspondence between the real number t and the point (x, y) on the unit circle when the unit circle is divided into 12 equal arcs.

t	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$7\pi/6$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$11\pi/6$
x												
y												

Example 1: Find the following:

(a) $\cos\dfrac{\pi}{3}$ (b) $\tan\dfrac{3\pi}{4}$ (c) $\csc\dfrac{7\pi}{6}$

III. Domain and Period of Sine and Cosine (Pages 268–269)

> ***What you should learn***
> How to use domain and period to evaluate sine and cosine functions and how to use a calculator to evaluate trigonometric functions

The sine function's domain is ____________________________,
and its range is __________________________.
The cosine function's domain is ___________________________,
and its range is __________________________.

The period of the sine function is __________________. The period of the cosine function is _________________.

Which trigonometric functions are even functions?

____

Which trigonometric functions are odd functions?

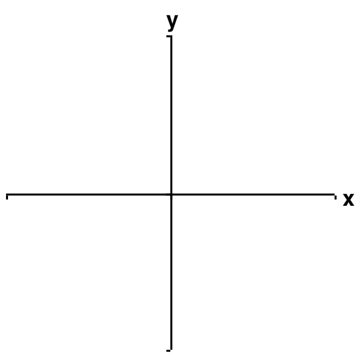
____

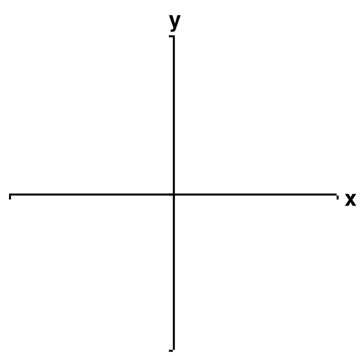
Example 2: Evaluate $\sin \frac{31\pi}{6}$

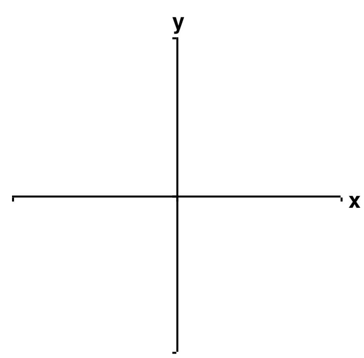
To evaluate the secant function with a calculator, _____________
____.

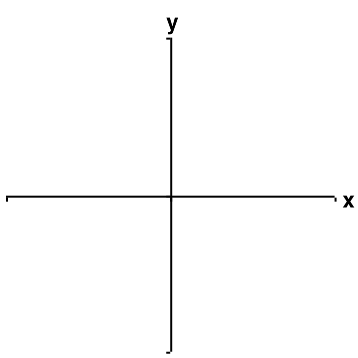
Example 3: Use a calculator to evaluate (a) $\tan 4\pi/3$, and (b) $\cos 3$.

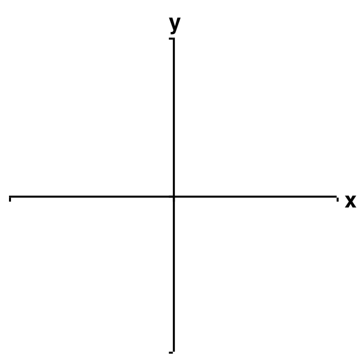
Additional notes

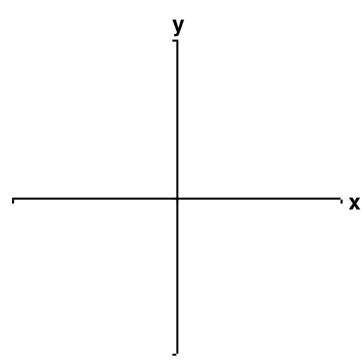












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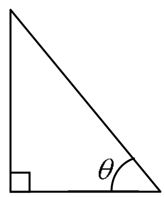
Section 4.3 Right Triangle Trigonometry

Objective: In this lesson you learned how to evaluate trigonometric functions of acute angles and how to use the fundamental trigonometric identities.

I. The Six Trigonometric Functions (Pages 273–275)

> ***What you should learn***
> How to evaluate trigonometric functions of acute angles and use a calculator to evaluate trigonometric functions

In the right triangle shown below, label the three sides of the triangle relative to the angle labeled θ as (a) the **hypotenuse,** (b) the **opposite side,** and (c) the **adjacent side.**



Let θ be an acute angle of a right triangle. Define the six trigonometric functions of the angle θ using opp = the length of the side opposite θ, adj = the length of the side adjacent to θ, and hyp = the length of the hypotenuse.

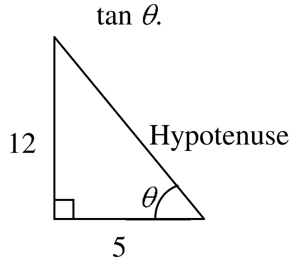
$\sin\theta =$ ____________ $\cos\theta =$ ____________

$\tan\theta =$ ____________ $\csc\theta =$ ____________

$\sec\theta =$ ____________ $\cot\theta =$ ____________

The cosecant function is the reciprocal of the ______________ function. The cotangent function is the reciprocal of the ________________ function. The secant function is the reciprocal of the ________________ function.

Example 1: In the right triangle below, find $\sin\theta$, $\cos\theta$, and $\tan\theta$.



Give the sines, cosines, and tangents of the following special angles:

$\sin 30° = \sin \frac{\pi}{6} =$ ________________________________

$\cos 30° = \cos \frac{\pi}{6} =$ ________________________________

$\tan 30° = \tan \frac{\pi}{6} =$ ________________________________

$\sin 45° = \sin \frac{\pi}{4} =$ ________________________________

$\cos 45° = \cos \frac{\pi}{4} =$ ________________________________

$\tan 45° = \tan \frac{\pi}{4} =$ ________________________________

$\sin 60° = \sin \frac{\pi}{3} =$ ________________________________

$\cos 60° = \cos \frac{\pi}{3} =$ ________________________________

$\tan 60° = \tan \frac{\pi}{3} =$ ________________________________

Cofunctions of complementary angles are ______________. If θ is an acute angle, then:

$\sin(90° - \theta) =$ __________ $\cos(90° - \theta) =$ __________

$\tan(90° - \theta) =$ __________ $\cot(90° - \theta) =$ __________

$\sec(90° - \theta) =$ __________ $\csc(90° - \theta) =$ __________

To use a calculator to evaluate trigonometric functions of angles measured in degrees, _________________________________

____.

Example 2: Use a calculator to evaluate (a) $\tan 35.4°$, and (b) $\cos 3.25°$

II. Trigonometric Identities (Pages 276–277)

> ***What you should learn***
> How to use the fundamental trigonometric identities

List six reciprocal identities:

1)

2)

3)

4)

5)

6)

List two quotient identities:

1)

2)

List three Pythagorean identities:

1)

2)

3)

III. Applications Involving Right Triangles (Pages 278–279)

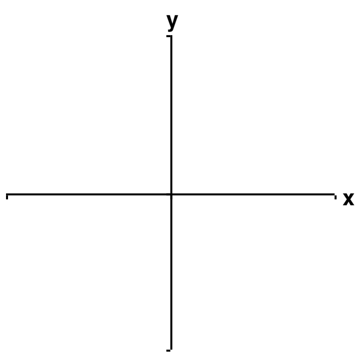
> ***What you should learn***
> How to use trigonometric functions to model and solve real-life problems

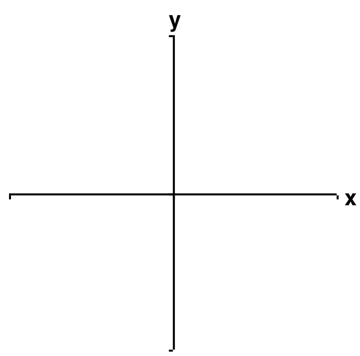
What does it mean to "solve a right triangle?"

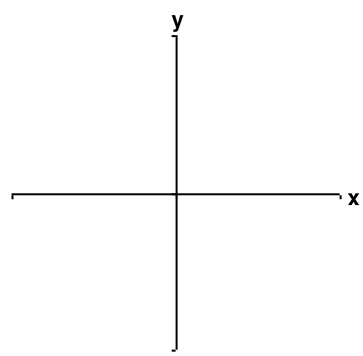
An **angle of elevation** is ________________________________
____.

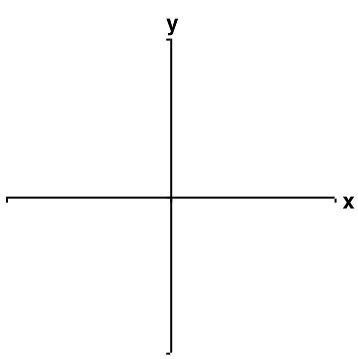
An **angle of depression** is ______________________________
____.

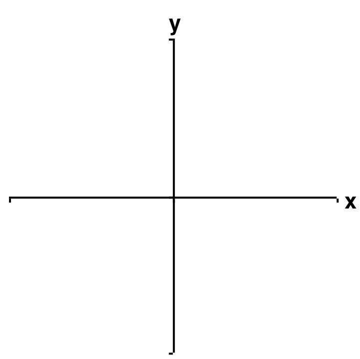
Describe a real-life situation in which solving a right triangle would be appropriate or useful.

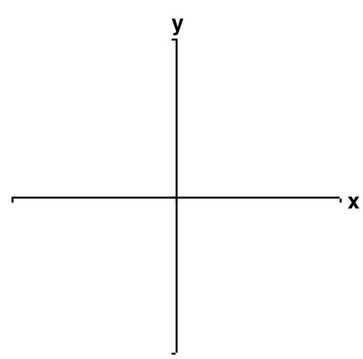












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Section 4.4 Trigonometric Functions of Any Angle

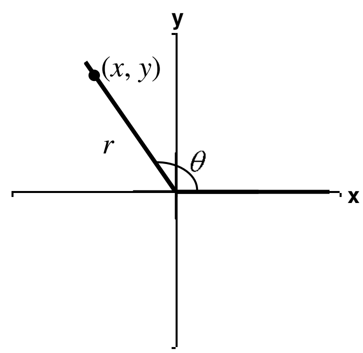
Objective: In this lesson you learned how to evaluate trigonometric functions of any angle.

Important Vocabulary	Define each term or concept.
Reference angles	

What you should learn
How to evaluate trigonometric functions of any angle

I. Introduction (Pages 284–285)

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. Complete the following definitions of the trigonometric functions of any angle:



$\sin\theta =$ _____________ $\cos\theta =$ _____________

$\tan\theta =$ _____________ $\cot\theta =$ _____________

$\sec\theta =$ _____________ $\csc\theta =$ _____________

Name the quadrants in which the sine function is positive.

____

Name the quadrants in which the sine function is negative.

____

Name the quadrants in which the cosine function is positive.

____

Name the quadrants in which the cosine function is negative.

____

Name the quadrants in which the tangent function is positive.

____

Name the quadrants in which the tangent function is negative.

____

Example 1: If $\sin\theta = \frac{1}{2}$ and $\tan\theta < 0$, find $\cos\theta$.

II. Reference Angles (Page 286)

> ***What you should learn***
> How to find reference angles

Example 2: Find the reference angle θ' for
(a) $\theta = 210°$ (b) $\theta = 4.1$

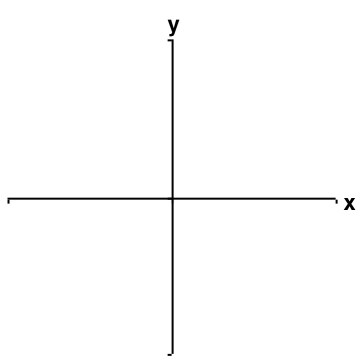
III. Trigonometric Functions of Real Numbers (Pages 287–288)

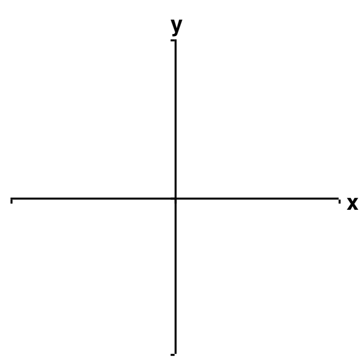
> ***What you should learn***
> How to evaluate trigonometric functions of real numbers

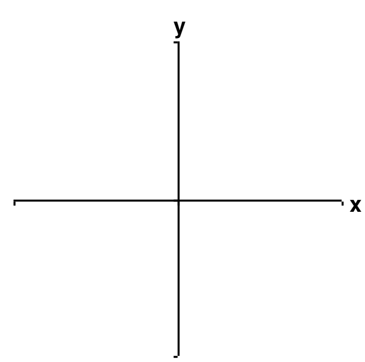
Describe how to find the value of a trigonometric function of any angle θ.

Example 3: Evaluate $\sin \frac{11\pi}{6}$.

Example 4: Evaluate $\cos 240°$.







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Section 4.5 Graphs of Sine and Cosine Functions

Objective: In this lesson you learned how to sketch the graphs of sine and cosine functions and translations of these functions.

Important Vocabulary	Define each term or concept.
Sine curve	
One cycle	
Amplitude	
Phase shift	

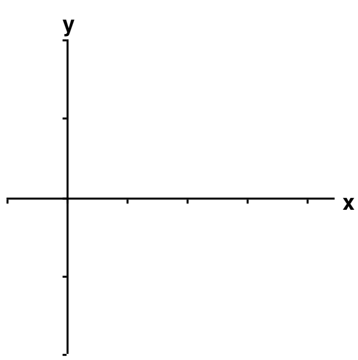
I. Basic Sine and Cosine Curves (Pages 292–293)

What you should learn
How to sketch the graphs of basic sine and cosine functions

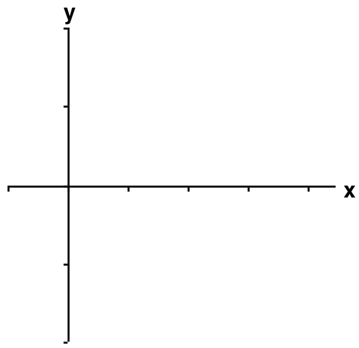
For $0 \le x \le 2\pi$, the sine function has its maximum point at _________________, its minimum point at _________________, and its intercepts at ______________________________________.

For $0 \le x \le 2\pi$, the cosine function has its maximum points at _______________________, its minimum point at ____________, and its intercepts at ____________________________________.

Example 1: Sketch the basic sine curve on the interval $[0, 2\pi]$.



Example 2: Sketch the basic cosine curve on the interval $[0, 2\pi]$.



II. Amplitude and Period of Sine and Cosine Curves
(Pages 294–295)

> ***What you should learn***
> How to use amplitude and period to help sketch the graphs of sine and cosine functions

The constant factor a in $y = a\sin x$ acts as _______________
____
______________________________________.

If $|a| > 1$, the basic sine curve is ____________________. If $|a| < 1$, the basic sine curve is ________________. The result is that the graph of $y = a\sin x$ ranges between ________________ instead of between -1 and 1. The absolute value of a is the _________________ of the function $y = a\sin x$.

The graph of $y = -0.5\sin x$ is a(n) ___________________ in the x-axis of the graph of $y = 0.5\sin x$.

Let b be a positive real number. The **period** of $y = a\sin bx$ and $y = a\cos bx$ is _________________. If $0 < b < 1$, the period of $y = a\sin bx$ is _________________ than 2π and represents a _____________________________ of the graph of $y = a\sin bx$.
If $b > 1$, the period of $y = a\sin bx$ is _______________ than 2π and represents a _____________________________ of the graph of $y = a\sin bx$.

Example 3: Find the amplitude and the period of $y = -4\cos 3x$.

Example 4: Find the five key points (intercepts, maximum points, and minimum points) of the graph of $y = -4\cos 3x$.

III. Translations of Sine and Cosine Curves (Pages 296–297)

> ***What you should learn***
> How to sketch translations of graphs of sine and cosine functions

The constant c in the general equations $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ creates ________________________________
____.

Comparing $y = a\sin bx$ with $y = a\sin(bx - c)$, the graph of $y = a\sin(bx - c)$ completes one cycle from ______________ to
________________. By solving for x, you can find the interval for one cycle is found to be ____________ to _______________.
This implies that the period of $y = a\sin(bx - c)$ is
_____________, and the graph of $y = a\sin(bx - c)$ is the graph of $y = a\sin bx$ shifted by the amount ______________.

Example 5: Find the amplitude, period, and phase shift of $y = 2\sin(x - \pi/4)$.

Example 6: Find the five key points (intercepts, maximum points, and minimum points) of the graph of $y = 2\sin(x - \pi/4)$.

The constant d in the equation $y = d + a\sin(bx - c)$ causes a(n)
_________________________. For $d > 0$, the shift is _________
____________. For $d < 0$, the shift is ______________________.
The graph oscillates about ______________________________.

IV. Mathematical Modeling (Page 298)

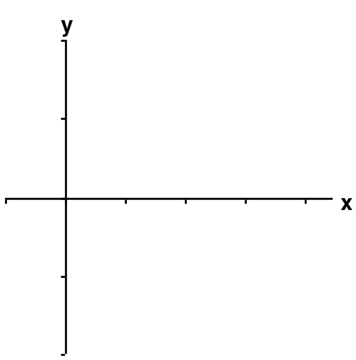
> ***What you should learn***
> How to use sine and cosine functions to model real-life data

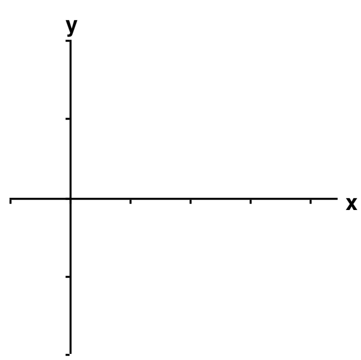
Describe a real-life situation which can be modeled by a sine or cosine function.

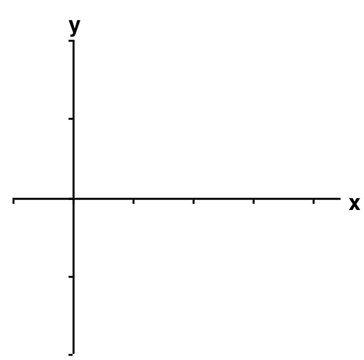
Example 7: Find a trigonometric function to model the data in the following table.

x	0	$\pi/2$	π	$3\pi/2$	2π
y	2	4	2	0	2

Additional notes







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Section 4.6 Graphs of Other Trigonometric Functions

Objective: In this lesson you learned how to sketch the graphs of other trigonometric functions.

Important Vocabulary	Define each term or concept.
Damping factor	

I. Graph of the Tangent Function (Pages 304–305)

What you should learn
How to sketch the graphs of tangent functions

Because the tangent function is odd, the graph of $y = \tan x$ is symmetric with respect to the _______________. The period of the tangent function is ____________. The tangent function has vertical asymptotes at $x =$ ________________ , where n is an integer. The domain of the tangent function is ______________________________, and the range of the tangent function is ______________________.

Describe how to sketch the graph of a function of the form $y = a\tan(bx - c)$.

II. Graph of the Cotangent Function (Page 306)

What you should learn
How to sketch the graphs of cotangent functions

The period of the cotangent function is ____________. The domain of the cotangent function is ____________________, and the range of the cotangent function is _________________.

The vertical asymptotes of the cotangent function occur at $x =$ ___________, where n is an integer.

III. Graphs of the Reciprocal Functions (Pages 307–308)

What you should learn
How to sketch the graphs of secant and cosecant functions

At a given value of x, the y-coordinate of csc x is the reciprocal of the y-coordinate of ________________.
The graph of $y = \csc x$ is symmetric with respect to the ________________. The period of the cosecant function is _____________. The cosecant function has vertical asymptotes at $x =$ ________________, where n is an integer. The domain of the cosecant function is ____________________, and the range of the cosecant function is ___________________.

At a given value of x, the y-coordinate of sec x is the reciprocal of the y-coordinate of ________________. The graph of $y = \sec x$ is symmetric with respect to the ________________.
The period of the secant function is _______. The secant function has vertical asymptotes, at $x =$ __________________.
The domain of the secant function is ______________________, and the range of the secant function is _____________________.

Describe how to sketch the graph of a secant or cosecant function.

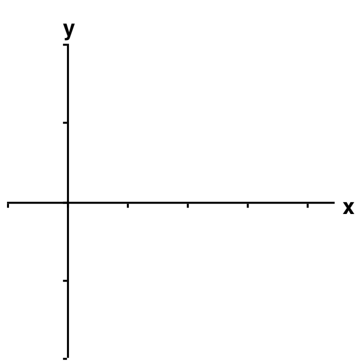
In comparing the graphs of the cosecant and secant functions with those of the sine and cosine functions, note that the "hills" and "valleys" are _____________________.

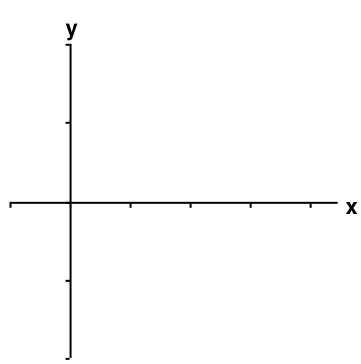
IV. Damped Trigonometric Graphs (Pages 309–310)

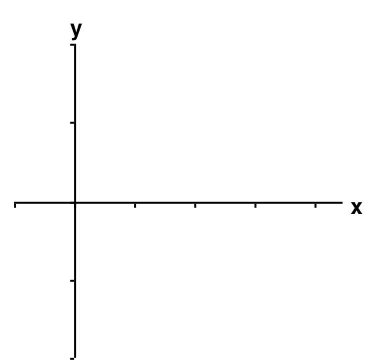
Explain how to sketch the graph of the damped trigonometric function $y = f(x)\cos x$, where $f(x)$ is the damping factor.

What you should learn
How to sketch the graphs of damped trigonometric functions

Additional notes







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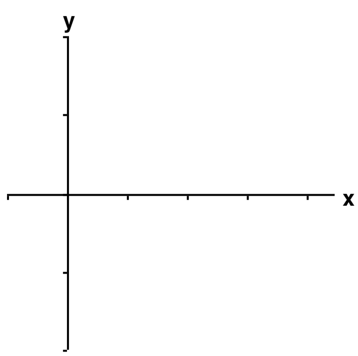

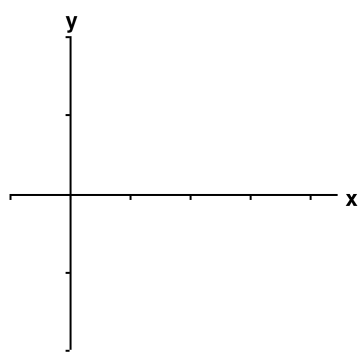

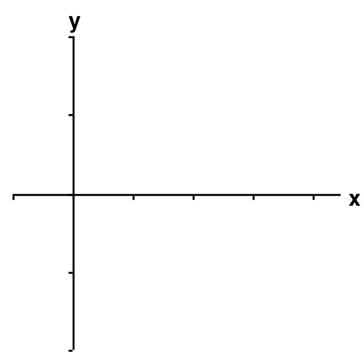

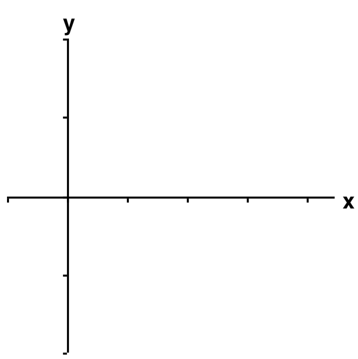

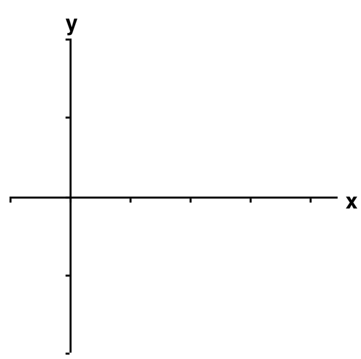

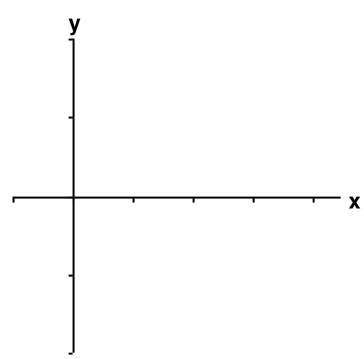

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Section 4.7 Inverse Trigonometric Functions

Objective: In this lesson you learned how to evaluate the inverse trigonometric functions and how to evaluate the composition of trigonometric functions.

I. Inverse Sine Function (Pages 315–316)

What you should learn How to evaluate and graph inverse sine functions

Give the definition of the **inverse sine function.**

The domain of $y = \arcsin x$ is ________________. The range of $y = \arcsin x$ is ______________________.

Example 1: Find the exact value: $\arcsin(-1)$.

II. Other Inverse Trigonometric Functions (Pages 317–319)

What you should learn How to evaluate and graph other inverse trigonometric functions

Give the definition of the **inverse cosine function.**

The domain of $y = \arccos x$ is ________________. The range of $y = \arccos x$ is ___________________.

Example 2: Find the exact value: $\arccos \frac{1}{2}$.

Give the definition of the **inverse tangent function.**

The domain of $y = \arctan x$ is ________________. The range of $y = \arctan x$ is ________________________.

Example 3: Find the exact value: $\arctan(\sqrt{3})$.

Example 4: Use a calculator to approximate the value (if possible). Round to four decimal places.
(a) arcos 0.85 (b) arcsin 3.1415

III. Compositions of Functions (Pages 320–321)

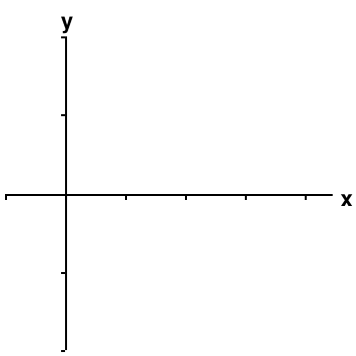
> ***What you should learn***
> How to evaluate compositions of trigonometric functions

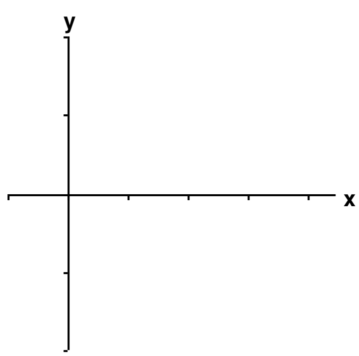
State the Inverse Property for the Sine function.

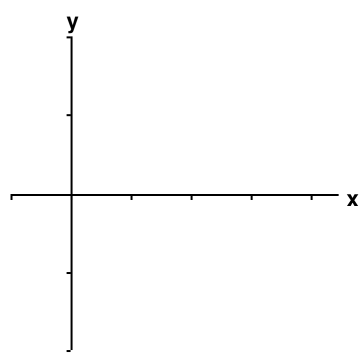
State the Inverse Property for the Cosine function.

State the Inverse Property for the Tangent function.

Example 5: If possible, find the exact value:
(a) arcsin(sin $3\pi/4$) (b) cos(arccos 0)







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Section 4.8 Applications and Models

Objective: In this lesson you learned how to use trigonometric functions to model and solve real-life problems.

I. Applications Involving Right Triangles (Pages 326–327)

> ***What you should learn***
> How to solve real-life problems involving right triangles

Example 1: A ladder leaning against a house reaches 24 feet up the side of the house. The ladder makes a 60° angle with the ground. How far is the base of the ladder from the house? Round your answer to two decimal places.

II. Trigonometry and Bearings (Page 328)

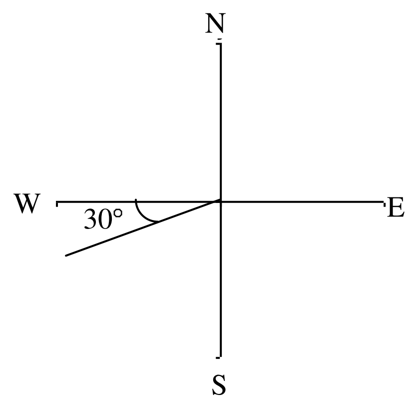
> ***What you should learn***
> How to solve real-life problems involving directional bearings

Used to give directions in surveying and navigation, a **bearing** measures ____

____.

The bearing N 70° E means ______________________________.

Example 2: Write the bearing for the path shown in the diagram at the right.



III. Harmonic Motion (Pages 329–331)

> ***What you should learn***
> How to solve real-life problems involving harmonic motion

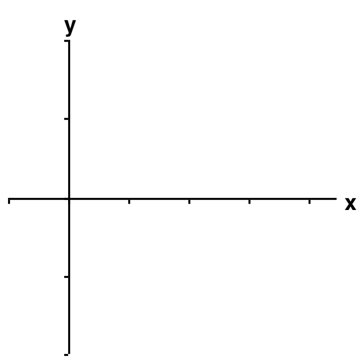
The vibration, oscillation, or rotation of an object under ideal conditions such that the object's uniform and regular motion can be described by a sine or cosine function is called ____________

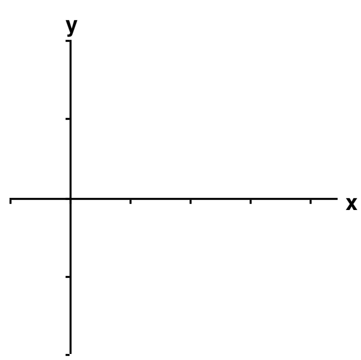
___________________.

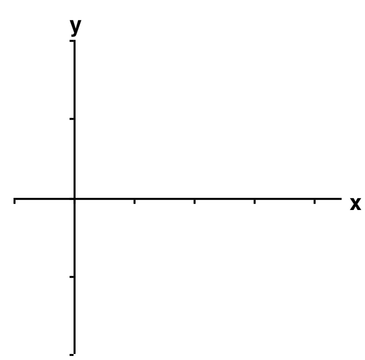
A point that moves on a coordinate line is said to be in **simple harmonic motion** if ____________________________________
____
____.

The simple harmonic motion has amplitude ___________, period
____________, and frequency __________________.

Example 3: Given the equation for simple harmonic motion $d = 3\sin\frac{t}{2}$, find:
(a) the maximum displacement,
(b) the frequency of the simple harmonic motion, and
(c) the period of the simple harmonic motion.







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Chapter 5 Analytic Trigonometry

Section 5.1 Using Fundamental Identities

Objective: In this lesson you learned how to use fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions.

What you should learn
How to recognize and write the fundamental trigonometric identities

I. Introduction (Page 350)

Name four ways in which the fundamental trigonometric identities can be used:

1)

2)

3)

4)

The Fundamental Trigonometric Identities

List six reciprocal identities:

1)

2)

3)

4)

5)

6)

List two quotient identities:

1)

2)

List three Pythagorean identities:

1)

2)

3)

List six cofunction identities:

1)

2)

3)

4)

5)

6)

List six even/odd identities:

1)

2)

3)

4)

5)

6)

II. Using the Fundamental Identities (Pages 351–353)

What you should learn
How to use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions

Example 1: Explain how to use the fundamental trigonometric identities to find the value of tan u given that $\sec u = 2$.

Example 2: Explain how to use the fundamental trigonometric identities to simplify $\sec x - \tan x \sin x$.

Example 3: Explain how to use a graphing utility to verify whether $\sec x \sin^3 x + \sin x \cos x = \tan x$ is an identity.

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Section 5.2 Verifying Trigonometric Identities

Objective: In this lesson you learned how to verify trigonometric identities.

I. Verifying Trigonometric Identities (Pages 357–361)

What you should learn
How to verify trigonometric identities

The key to both verifying identities and solving equations is ____
____
____.

An identity is ____
____.

Complete the following list of guidelines for verifying trigonometric identities:

1)

2)

3)

4)

5)

Example 1: Describe a strategy for verifying the identity $\sin\theta\tan\theta+\cos\theta=\sec\theta$. Then verify the identity.

Example 2: Describe a strategy for verifying the identity $\sin^2 x(\csc x - 1)(\csc x + 1) = 1 - \sin^2 x$. Then verify the identity.

Example 3: Verify the identity $\cot^5 \alpha = \cot^3 \alpha \csc^2 \alpha - \cot^3 \alpha$.

Additional notes

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Name ____ Date ___________

Section 5.3 Solving Trigonometric Equations

Objective: In this lesson you learned how to use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations.

I. Introduction (Pages 365–367)

> ***What you should learn***
> How to use standard algebraic techniques to solve trigonometric equations

To solve a trigonometric equation, ________________________
____.

The preliminary goal in solving trigonometric equations is _____
____.

How many solutions does the equation $\sec x = 2$ have? Explain.

Example 1: Solve $2\cos^2 x - 1 = 0$.

To solve an equation in which two or more trigonometric functions occur, ____
____
_________________________________.

II. Equations of Quadratic Type (Pages 368–369)

> ***What you should learn***
> How to solve trigonometric equations of quadratic type

Give an example of a trigonometric equation of quadratic type.

To solve a trigonometric equation of quadratic type,
____
____.

Example 2: Solve $\tan^2 x + 2\tan x = -1$.

Care must be taken when squaring each side of a trigonometric equation to obtain a quadratic because ______________________
____
____
____.

III. Functions Involving Multiple Angles (Page 370)

What you should learn
How to solve trigonometric equations involving multiple angles

Give an example of a trigonometric function of multiple angles.

Example 3: Solve $\sin 4x = \dfrac{\sqrt{2}}{2}$.

IV. Using Inverse Functions (Page 371–372)

What you should learn
How to use inverse trigonometric functions to solve trigonometric equations

Example 4: Use inverse functions to solve the equation $\tan^2 x + 4\tan x + 4 = 0$.

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Section 5.4 Sum and Difference Formulas

Objective: In this lesson you learned how to use sum and difference formulas to rewrite and evaluate trigonometric functions.

I. Using Sum and Difference Formulas (Pages 377–380)

What you should learn
How to use sum and difference formulas to evaluate trigonometric functions, verify identities, and solve trigonometric equations

List the sum and difference formulas for sine, cosine, and tangent.

Example 1: Use a sum or difference formula to find the exact value of tan 255°.

Example 2: Find the exact value of cos 95° cos 35° + sin 95° sin 35°.

A **reduction formula** is ________________________________
____
____.

Example 3: Derive a reduction formula for $\sin\left(t + \dfrac{\pi}{2}\right)$.

Example 4: Find all solutions of $\cos(x - \frac{\pi}{3}) + \cos(x + \frac{\pi}{3}) = 1$ in the interval $[0, 2\pi)$.

Additional notes

Homework Assignment

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Exercises

Name ____ Date ____________

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

Objective: In this lesson you learned how to use multiple-angle formulas, power-reducing formulas, half-angle formulas, and product-to-sum formulas to rewrite and evaluate trigonometric functions.

I. Multiple-Angle Formulas (Pages 384–385)

> ***What you should learn***
> How to use multiple-angle formulas to rewrite and evaluate trigonometric functions

The most commonly used multiple-angle formulas are the ______________________________, which are listed below:

$\sin 2u =$ ______________________

$\cos 2u =$ ________________________

$=$ ________________________

$=$ ________________________

$\tan 2u =$ __________________________

To obtain other multiple-angle formulas, ___________________

____

____

____.

Example 1: Use multiple-angle formulas to express $\cos 3x$ in terms of $\cos x$.

II. Power-Reducing Formulas (Page 386)

> ***What you should learn***
> How to use power-reducing formulas to rewrite and evaluate trigonometric functions

The double-angle formulas can be used to obtain the

____.

The power-reducing formulas are:

$\sin^2 u =$ ______________________

$\cos^2 u =$ ______________________

$\tan^2 u =$ _______________________________

III. Half-Angle Formulas (Page 387)

> ***What you should learn***
> How to use half-angle formulas to rewrite and evaluate trigonometric functions

List the **half-angle formulas:**

$\sin \frac{u}{2} =$ _______________________

$\cos \frac{u}{2} =$ _______________________

$\tan \frac{u}{2} =$ _____________________ = _____________________

The signs of sin (u/2) and cos (u/2) depend on ______________

____.

Example 2: Find the exact value of tan 15°.

IV. Product-to-Sum Formulas (Pages 388–389)

> ***What you should learn***
> How to use product-to-sum and sum-to-product formulas to rewrite and evaluate trigonometric functions

The **product-to-sum formulas** are used in calculus to ________

____

____.

The product-to-sum formulas are:

$\sin u \sin v =$ ________________________________

$\cos u \cos v =$ ________________________________

$\sin u \cos v =$ ________________________________

$\cos u \sin v =$ ________________________________

Example 3: Write $\cos 3x \cos 2x$ as a sum or difference.

The **sum-to-product formulas** can be used to ______________

____.

The sum-to-product formulas are:

$\sin u + \sin v =$ ____________________________________

$\sin u - \sin v =$ ____________________________________

$\cos u + \cos v =$ ____________________________________

$\cos u - \cos v =$ _____________________________________

Example 4: Write $\cos 4x + \cos 2x$ as a sum or difference.

Additional notes

Additional notes

Homework Assignment

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Name ____ Date ____________

Chapter 6 Additional Topics in Trigonometry

Section 6.1 Law of Sines

Objective: In this lesson you learned how to use the Law of Sines to solve oblique triangles and how to find the areas of oblique triangles.

Important Vocabulary	Define each term or concept.
Oblique triangle	

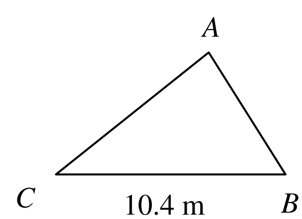
I. Introduction (Pages 404–405)

What you should learn
How to use the Law of Sines to solve oblique triangles (AAS or ASA)

State the **Law of Sines.**

To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle. Describe two cases that can be solved using the Law of Sines.

Example 1: For the triangle shown at the right, $A = 31.6°$, $C = 42.9°$, and $a = 10.4$ meters. Find the length of side c.



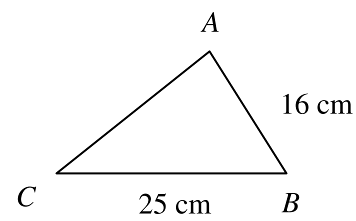
II. The Ambiguous Case (SSA) (Pages 406–407)

What you should learn
How to use the Law of Sines to solve oblique triangles (SSA)

If two sides and one opposite angle of an oblique triangle are given, _______________ possible situations can occur, which are:

Example 2: For a triangle having $A = 25°$, $b = 54$ feet, and $a = 26$ feet, how many solutions are possible?

Example 3: For the triangle shown at the right, $A = 110°$, $c = 16$ centimeters, and $a = 25$ centimeters. Find the length of side b.



III. Area of an Oblique Triangle (Pages 408–409)

What you should learn
How to find areas of oblique triangles and use the Law of Sines to model and solve real-life problems

The area of any triangle is ________________ the product of the lengths of two sides times the sine of ______________________.
That is,

Area = ____

Example 4: Find the area of a triangle having two sides of lengths 30 feet and 48 feet and an included angle of 40°.

Describe a real-life situation in which the Law of Sines could be used.

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Name ____ Date ____________

Section 6.2 Law of Cosines

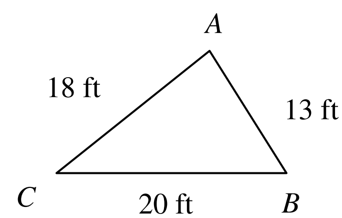
Objective: In this lesson you learned how to use the Law of Cosines to solve oblique triangles and to use Heron's Formula to find the area of a triangle.

I. Introduction (Pages 413–414)

State the **Law of Cosines**.

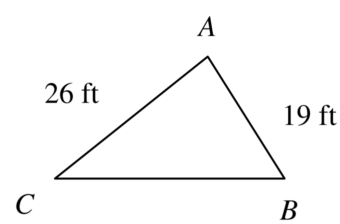
What you should learn
How to use the Law of Cosines to solve oblique triangles (SSS or SAS)

Example 1: Using the triangle shown at the right, find angle A.



When given the lengths of all three sides of a triangle and asked to find all three angles, which angle should be found first? Why?

Example 2: In the triangle shown at the right, if $A = 62°$, find the length of side a.



II. Applications (Page 415)

Describe a real-life situation in which the Law of Cosines could be used.

What you should learn
How to use the Law of Cosines to model and solve real-life problems

III. Heron's Area Formula (Page 416)

> ***What you should learn***
> How to use Heron's Area Formula to find areas of triangles

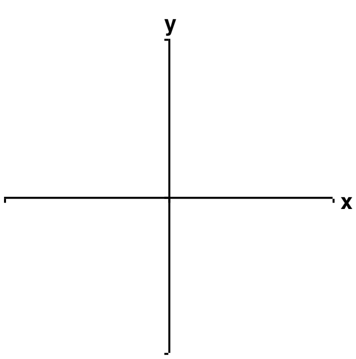
Heron's Area Formula states that given any triangle with sides of length a, b, and c, the area of the triangle is:

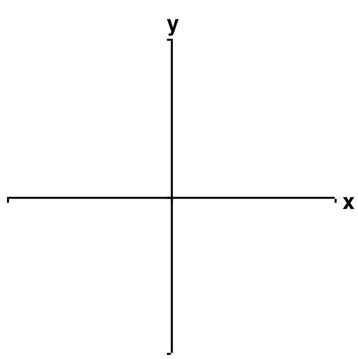
Area = $\sqrt{\qquad\qquad\qquad}$ ________________________________

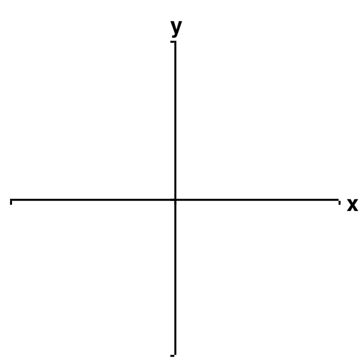
where s = ____________________.

Example 3: Find the area of a triangle having sides of length $a = 14$ cm, $b = 21$ cm, and $c = 27$ cm.

Additional notes







Homework Assignment

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Exercises

Name ____ Date ____________

Section 6.3 Vectors in the Plane

Objective: In this lesson you learned how to represent vectors as directed line segments, perform mathematical operations on vectors, and find direction angles of vectors.

Important Vocabulary	Define each term or concept.
Vector v in the plane	
Standard position	
Zero vector	
Unit vector	
Standard unit vectors	
Direction angle	

I. Introduction (Page 420)

> ***What you should learn***
> How to represent vectors as directed line segments

A **directed line segment** $\overrightarrow{PQ}$, has ____________________ P and ______________________ Q.

The **magnitude,** or ________________, of the directed line segment $\overrightarrow{PQ}$, is denoted by ________________ and can be found by ____.

II. Component Form of a Vector (Page 421)

> ***What you should learn***
> How to write the component forms of vectors

A vector whose initial point is at the origin (0, 0) can be uniquely represented by the coordinates of its terminal point (v_1, v_2). This is the ____, written $\mathbf{v} = \langle v_1, v_2 \rangle$, where v_1 and v_2 are the __________________ of **v**.

The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is
$\overrightarrow{PQ} =$ ______________________ = _____________ = **v**.

The **magnitude** (or length) of **v** is:

$\|\mathbf{v}\| = \sqrt{\rule{6cm}{0.4pt}} = \sqrt{\rule{3cm}{0.4pt}}$

Example 1: Find the component form and magnitude of the vector **v** that has (1, 7) as its initial point and (4, 3) as its terminal point.

III. Vector Operations (Pages 422–423)

> ***What you should learn***
> How to perform basic vector operations and represent vectors graphically

Geometrically, the product of a vector **v** and a scalar k is ______
____.

If k is positive, k**v** has the _____________ direction as **v**, and if k is negative, k**v** has the _____________ direction.

To add two vectors **u** and **v** geometrically, _________________
____
____
____.

This technique is called the ___________________________ for vector addition because the vector **u** + **v**, often called the __________________ of vector addition, is _________________
____.

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a scalar (a real number). The **sum** of **u** and **v,** is the vector

u + **v** = _______________________

The **scalar multiple** of k times **u,** is the vector:

k**u** = ____________________________

Example 2: Let $\mathbf{u} = \langle 1, 6 \rangle$ and $\mathbf{v} = \langle -4, 2 \rangle$. Find:
(a) 3**u**
(b) **u** + **v**

Let **u**, **v**, and **w** be vectors and c and d be scalars. Complete the following properties of vector addition and scalar multiplication:

1. $\mathbf{u} + \mathbf{v} =$ ________________
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} =$ ______________________
3. $\mathbf{u} + \mathbf{0} =$ _____________
4. $\mathbf{u} + (-\mathbf{u}) =$ _____________
5. $c(d\mathbf{u}) =$ _________________
6. $(c + d)\mathbf{u} =$ _________________
7. $c(\mathbf{u} + \mathbf{v}) =$ _________________
8. $1(\mathbf{u}) =$ _____________ , $0(\mathbf{u}) =$ _____________
9. $\|c\mathbf{v}\| =$ ____________________

IV. Unit Vectors (Pages 424–425)

> ***What you should learn***
> How to write vectors as linear combinations of unit vectors

To find a unit vector **u** that has the same direction as a given nonzero vector **v**, ____

____.

In this case, the vector **u** is called a _________________________

_____________________.

Example 3: Find a unit vector in the direction of $\mathbf{v} = \langle -8, 6\rangle$.

Let $\mathbf{v} = \langle v_1, v_2\rangle$. Then the standard unit vectors can be used to represent **v** as $\mathbf{v} =$ _________________ , where the scalar v_1 is called the _________________________________ and the scalar v_2 is called the _______________________________. The vector sum $v_1\mathbf{i} + v_2\mathbf{j}$ is called a __________________________ of the vectors **i** and **j**.

Example 4: Let $\mathbf{v} = \langle -5, 3\rangle$. Write **v** as a linear combination of the standard unit vectors **i** and **j**.

Example 5: Let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 9\mathbf{j}$. Find $\mathbf{v} + \mathbf{w}$.

V. Direction Angles (Page 426)

What you should learn
How to find the direction angles of vectors

If **u** is a unit vector and θ is the angle (measured counterclockwise) from the positive x-axis to **u**, the terminal point of **u** lies on the unit circle and

$\mathbf{u} = \langle x, y \rangle =$ ____________________ = ______________________

Now, if $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ is any vector that makes an angle θ with the positive x-axis, then it has the same direction as **u** and

$\mathbf{v} =$ ________________________ = _________________________

Because $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, then the direction angle θ for **v** can be determined from $\tan \theta =$ _____________.

Example 6: Let $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$. Find the direction angle for **v**.

VI. Applications of Vectors (Pages 427–428)

What you should learn
How to use vectors to model and solve real-life problems

Describe several real-life applications of vectors.

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Section 6.4 Vectors and Dot Products

Objective: In this lesson you learned how to find the dot product of two vectors and use properties of the dot product.

Important Vocabulary	Define each term or concept.
Angle between two nonzero vectors	
Orthogonal vectors	
Vector components	

I. The Dot Product of Two Vectors (Page 434)

What you should learn
How to find the dot product of two vectors and use the properties of the dot product

The **dot product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

______________________. This product yields a __________.

Let **u**, **v**, and **w** be vectors in the plane or in space and let c be a scalar. Complete the following properties of the dot product:

1. $\mathbf{u} \bullet \mathbf{v} =$ _______________
2. $\mathbf{0} \bullet \mathbf{v} =$ ____________
3. $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) =$ ____________________
4. $\mathbf{v} \bullet \mathbf{v} =$ _______________
5. $c(\mathbf{u} \bullet \mathbf{v}) =$ _______________ = _______________

Example 1: Find the dot product: $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$.

II. The Angle Between Two Vectors (Pages 435–436)

What you should learn
How to find the angle between two vectors and determine whether two vectors are orthogonal

If θ is the angle between two nonzero vectors **u** and **v**, then

_____________________________________.

Example 2: Find the angle between $\mathbf{v} = \langle 5, -4 \rangle$ and $\mathbf{w} = \langle 9, -2 \rangle$.

An alternative way to calculate the dot product between two vectors **u** and **v**, given the angle θ between them, is

______________________________.

Two vectors **u** and **v** are orthogonal if ________________.

Example 3: Are the vectors $\mathbf{u} = \langle 1, -4 \rangle$ and $\mathbf{v} = \langle 6, 2 \rangle$ orthogonal?

III. Finding Vector Components (Pages 437–438)

What you should learn
How to write vectors as the sums of two vector components

Let **u** and **v** be nonzero vectors such that $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$, where $\mathbf{w}_1$ and $\mathbf{w}_2$ are orthogonal and $\mathbf{w}_1$ is parallel to (or a scalar multiple of) **v**. The vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ are called ____________________ ____________. The vector $\mathbf{w}_1$ is the **projection** of **u** onto **v** and is denoted by ____________________. The vector $\mathbf{w}_2$ is given by ____________________ .

Let **u** and **v** be nonzero vectors. The projection of **u** onto **v** is given by $\text{proj}_{\mathbf{v}}\,\mathbf{u}$ = ______________________________.

IV. Work (Page 439)

What you should learn
How to use vectors to find the work done by a force

The work W done by a constant force **F** as its point of application moves along the vector $\overrightarrow{PQ}$ is given by either of the following:

1.

2.

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Section 6.5 Trigonometric Form of a Complex Number

Objective: In this lesson you learned how to multiply and divide complex numbers written in trigonometric form and how to find powers and *n*th roots of complex numbers.

Important Vocabulary	Define each term or concept.
***n*th roots of unity**	

I. The Complex Plane (Page 443)

What you should learn
How to plot complex numbers in the complex plane and find absolute values of complex numbers

The **absolute value of the complex number** $a + bi$ is defined as

____.

The absolute value of the complex number $z = a + bi$ is given by $|a + bi| = \sqrt{\_\_\_\_\_\_\_\_}$.

II. Trigonometric Form of a Complex Number (Pages 444–445)

What you should learn
How to write trigonometric forms of complex numbers

The **trigonometric form of the complex number** $z = a + bi$ is

$z =$ ________________________,

where $a =$ ________________,

$b =$ ________________,

$r = \sqrt{\_\_\_\_\_\_\_\_}$, and

$\tan\theta =$ ______________.

The number r is the ________________ of z, and θ is called an ________________ of z.

The trigonometric form of a complex number is also called the

__________________.

III. Multiplication and Division of Complex Numbers (Pages 446–447)

> ***What you should learn***
> How to multiply and divide complex numbers written in trigonometric form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be complex numbers. Then:

$z_1z_2 =$ ____________________________________

$z_1/z_2 =$ ____

Describe how to multiply two complex numbers.

Describe how to divide two complex numbers.

IV. Powers of Complex Numbers (Page 448)

> ***What you should learn***
> How to use DeMoivre's Theorem to find powers of complex numbers

State **DeMoivre's Theorem**.

IV. Roots of Complex Numbers (Pages 449–451)

> ***What you should learn***
> How to find *n*th roots of complex numbers

The complex number $u = a + bi$ is an ***n*th root** of the complex number z if ______________________________.

For a positive integer n, the complex number $z = r(\cos\theta + i\sin\theta)$ has ____ given by $\sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)$, where $k = 0, 1, 2, \ldots, n - 1$.

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Chapter 7 Linear Systems and Matrices

Section 7.1 Solving Systems of Equations

Objective: In this lesson you learned how to solve systems of equations by substitution and by graphing and how to use systems of equations to model and solve real-life problems.

Important Vocabulary Define each term or concept.

Systems of equations

Solution of a system of equations (in two variables)

Method of substitution

Point of intersection

Break-even point

I. The Methods of Substitution and Graphing
(Pages 470–474)

What you should learn
How to use the methods of substitution and graphing to solve systems of equations in two variables

To check that the ordered pair (– 3, 4) is the solution of a system of two equations, ______________________________
______________________________.

List the steps necessary for solving a system of two equations in x and y using the method of substitution.

The solution of a system of equations corresponds to the

____ of the

graphs of the equations in the system.

List the necessary steps for using the method of graphing to solve a system of two equations in x and y.

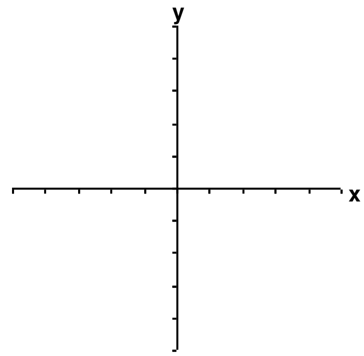
Explain what is meant by back-substitution.

Example 1: Solve the system of equations using the method of substitution.

$$\begin{cases} 2x + y = 2 \\ x - 2y = -9 \end{cases}$$

Example 2: Solve the system of equations using the method of graphing.

$$\begin{cases} x^2 - y = 5 \\ -x + y = -3 \end{cases}$$



II. Application (Page 475)

The total cost C of producing x units of a product typically has two components: ____.

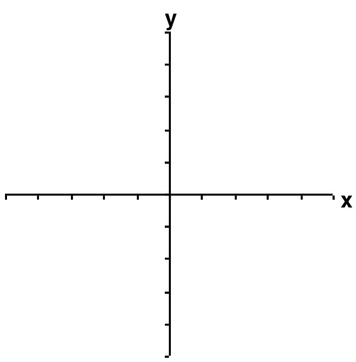
In break-even analysis, the break-even point corresponds to the ____________________________________ of the cost and revenue curves.

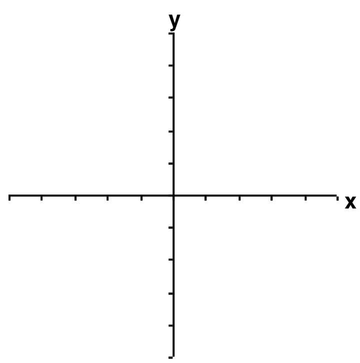
Break-even analysis can also be approached from the point of view of profit. In this case, consider the profit function, which is __________________. The break-even point occurs when profit equals ___________.

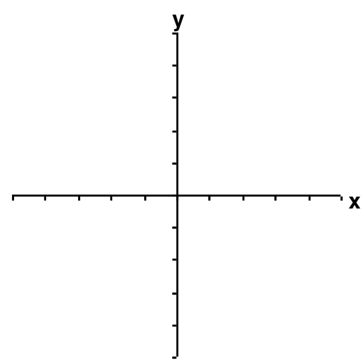
What you should learn
How to use systems of equations to model and solve real-life problems

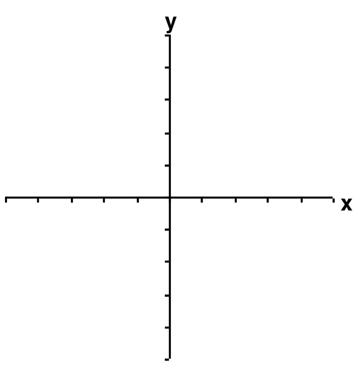
Example 3: The cost of producing x units is $C = 1.5x + 15{,}000$ and the revenue obtained by selling x units is $R = 5x$. How many items should be sold to break even?

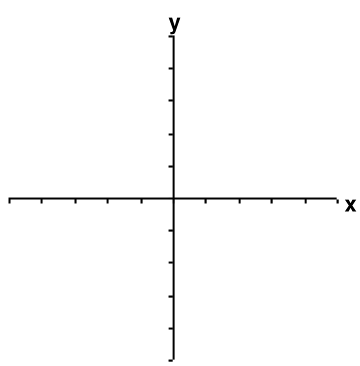
Additional notes

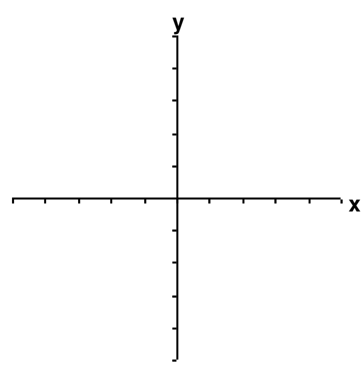












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Section 7.2 Systems of Linear Equations in Two Variables

Objective: In this lesson you learned how to solve a system of equations by elimination and how to use systems of equations to model and solve real-life problems.

Important Vocabulary Define each term or concept.

Method of elimination

Equivalent systems

Consistent system

Inconsistent system

I. The Method of Elimination (Pages 480–481)

What you should learn
How to use the method of elimination to solve systems of linear equations in two variables

List the steps necessary for solving a system of two linear equations in x and y using the method of elimination.

The operations that can be performed on a system of linear equations to produce an equivalent system are:

(1)

(2)

(3)

Example 1: Describe a strategy for solving the system of linear equations using the method of elimination.

$$\begin{cases} 3x + y = 9 \\ 4x - 2y = -1 \end{cases}$$

Example 2: Solve the system of linear equations using the method of elimination.

$$\begin{cases} 4x + y = -3 \\ x - 3y = 9 \end{cases}$$

II. Graphical Interpretation of Two-Variable Systems
(Pages 482–483)

> ***What you should learn***
> How to graphically interpret the number of solutions of a system of linear equations in two variables

If a system of linear equations has two different solutions, it must have ________________________________ solutions.

For a system of two linear equations in two variables, list the possible number of solutions the system can have and give a graphical interpretation of the solutions.

If a false statement such as 9 = 0 is obtained while solving a system of linear equations using the method of elimination, then the system has ________________________.

If a statement that is true for all values of the variables, such as 0 = 0, is obtained while solving a system of linear equations using the method of elimination, then the system has

____.

Example 3: Is the following system consistent or inconsistent?
How many solutions does the system have?

$$\begin{cases} x - 3y = 2 \\ -4x + 12y = 8 \end{cases}$$

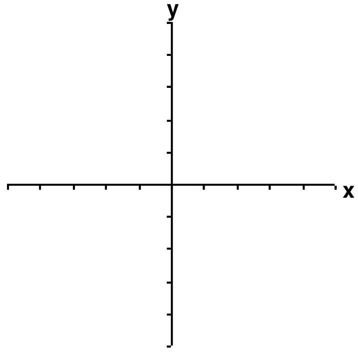
III. Application (Page 484)

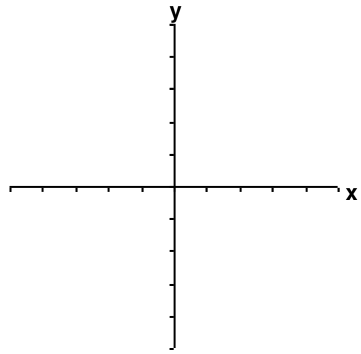
What you should learn
How to use systems of linear equations in two variables to model and solve real-life problems

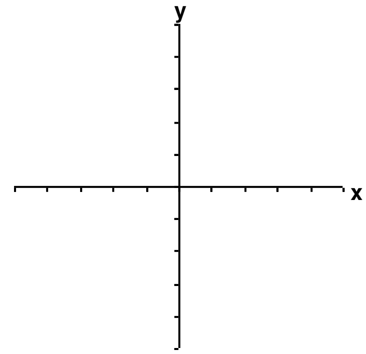
When may a system of linear equations be an appropriate mathematical model for solving a real-life application?

Give an example of a real-life application that could be solved with a system of linear equations.

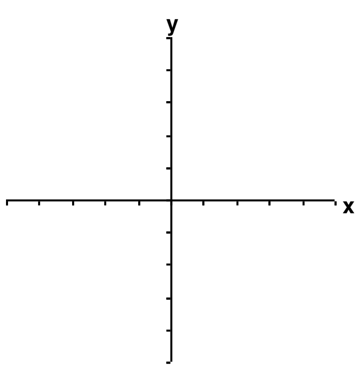
Additional notes

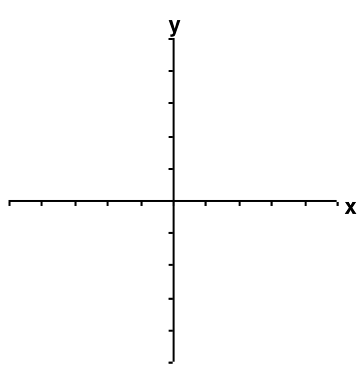


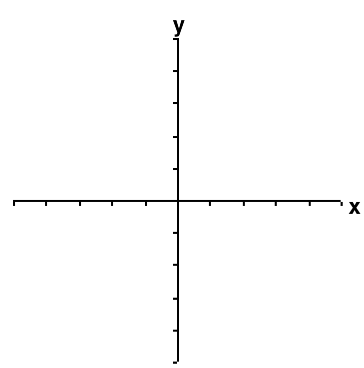




Additional notes







Homework Assignment

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Section 7.3 Multivariable Linear Systems

Objective: In this lesson you learned how to solve a system of equations by Gaussian elimination, how to recognize linear systems in row-echelon form and to use back substitution to solve the system, how to solve nonsquare systems of equations, and how to use a system of equations to model and solve real-life problems.

Important Vocabulary Define each term or concept.

Row-echelon form

Gaussian elimination

Nonsquare system of equations

Graph of an equation in three variables

Partial fraction

Partial fraction decomposition

I. Row-Echelon Form and Back-Substitution (Page 489)

When elimination is used to solve a system of linear equations, the goal is ____

____.

> ***What you should learn***
> How to use back-substitution to solve linear systems in row-echelon form

Example 1: Solve the system of linear equations.

$$\begin{cases} x + y - z = 9 \\ \quad\; y - 2z = 4 \\ \qquad\quad\; z = 1 \end{cases}$$

II. Gaussian Elimination (Pages 490–492)

To solve a system that is not in row-echelon form, ___________

____

____.

> ***What you should learn***
> How to use Gaussian elimination to solve systems of linear equations

List the three elementary row operations that can be used on a system of linear equations to produce an equivalent system of linear equations.

1.
2.
3.

The number of solution(s) of a system of linear equations in more than two variables must fall into one of the following three categories:

1.
2.
3.

Example 2: Solve the system of linear equations.

$$\begin{cases} x + y + z = 3 \\ 2x - y + 3z = 16 \\ x - 2y - z = 1 \end{cases}$$

A consistent system with exactly one solution is

____________________. A consistent system with infinitely

many solutions is ___________________.

Example 3: The following equivalent system is obtained during the course of Gaussian elimination. Write the solution of the system.

$$\begin{cases} x + 2y - z = 4 \\ y + 2z = 8 \\ 0 = 0 \end{cases}$$

III. Nonsquare Systems (Page 493)

What you should learn
How to solve nonsquare systems of linear equations

In a square system of linear equations, the number of equations

in the system is ___________________ the number of variables.

A system of linear equations cannot have a unique solution unless there are ____

____.

Example 4: Solve the system of linear equations.

$$\begin{cases} x + y + z = 1 \\ x - 2y - 2z = 4 \end{cases}$$

IV. Graphical Interpretation of Three-Variable Systems (Page 494)

What you should learn
How to graphically interpret three-variable linear systems

To construct a **three-dimensional coordinate system,** ________

____

____.

To sketch the graph of a plane, _____________________________

____

____

____.

The graph of a system of three linear equations in three variables consists of _______________ planes. When these planes intersect in a single point, the system has _________________ solution(s). When the planes have no point in common, the system has _____________ solution(s). When the planes intersect in a line or a plane, the system has __________________ solution(s).

V. Partial Fraction Decomposition (Pages 495–497)

What you should learn
How to use systems of linear equations to write partial fraction decompositions of rational expressions

Suppose the rational expression $N(x)/D(x)$ is an improper fraction. Before the expression can be decomposed into partial fractions, you must _______________________________________

____

____

____

____.

To decompose a proper rational expression into partial fractions, completely factor the denominator into factors of the form

__________________ and ______________________, where

______________________ is irreducible.

Describe how to deal with both linear factors and quadratic factors in the next step of a partial fraction decomposition.

To find the **basic equation** of a partial fraction decomposition, ____

____.

To solve the basic equation, _____________________________

____

____.

Example 5: Write the form of the partial fraction decomposition for $\dfrac{x-4}{x^2-8x+12}$.

Example 6: Solve the basic equation $5x+3 = A(x-1)+B(x+3)$ for A and B.

Homework Assignment

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Exercises

Name ____ Date ____________

Section 7.4 Matrices and Systems of Equations

Objective: In this lesson you learned how to write matrices, identify their dimensions, and perform elementary row operations and how to use Gaussian elimination and Gauss-Jordan elimination with matrices to solve systems of linear equations.

Important Vocabulary Define each term or concept.

Entry of a matrix

Dimension of a matrix

Square matrix

Main diagonal

Row matrix

Column matrix

Elementary row operations

Gauss-Jordan elimination

I. Matrices (Pages 504–505)

What you should learn
How to write matrices and identify their dimensions

If m and n are positive integers, define an $m \times n$ **matrix**.

An $m \times n$ matrix has ___________ rows and ___________ columns.

An **augmented matrix** is ________________________________
____
____.

A **coefficient matrix** is ________________________________

____

____.

Example 1: Consider the following system of equations.

$$\begin{cases} 2x + y - z = 5 \\ x - 3y + 2z = 9 \\ 3x + 2y = 1 \end{cases}$$

(a) Write the augmented matrix for this system.
(b) What is the dimension of the augmented matrix?
(c) Write the coefficient matrix for this system.
(d) What is the dimension of the coefficient matrix?

II. Elementary Row Operations (Page 506)

> ***What you should learn***
> How to perform elementary row operations on matrices

Two matrices are **row-equivalent** if ________________

____

____.

The **elementary row operations** on a matrix are:

III. Gaussian Elimination with Back-Substitution (Pages 507–510)

> ***What you should learn***
> How to use matrices and Gaussian elimination to solve systems of linear equations

A matrix in **row-echelon form** has the following three properties:

1.

2.

3.

A matrix in row-echelon form is in **reduced row-echelon form** if ____

____.

List the steps for solving a system of linear equations using Gaussian Elimination with Back-Substitution.

If, during the elimination process, you obtain a row with zeros except for the last entry, you can conclude that the system is ________________________.

Example 2: Solve the following system using Gaussian Elimination with Back-Substitution.

$$\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 1 \\ x - 3y + 5z = -11 \end{cases}$$

IV. Gauss-Jordan Elimination (Pages 511–512)

What you should learn
How to use matrices and Gauss-Jordan elimination to solve systems of linear equations

Example 3: Apply Gauss-Jordan elimination to the following matrix to obtain the unique reduced row-echelon form of the matrix.

$$\left[\begin{array}{ccc:c} 1 & 4 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Example 4: Solve the following system using Gauss-Jordan elimination.

$$\begin{cases} 2x - y + 3z = 1 \\ x + 2y - 4z = -6 \\ -2x + 3y - z = 13 \end{cases}$$

Homework Assignment

Page(s)

Exercises

Name ____ Date ____________

Section 7.5 Operations with Matrices

Objective: In this lesson you learned how to perform operations with matrices.

Important Vocabulary Define each term or concept.

Scalar multiple

Zero matrix

Additive identity

Matrix multiplication

Identity matrix of dimension *n* × *n*

I. Equality of Matrices (Page 518)

What you should learn
How to decide whether two matrices are equal

Name three ways that a matrix may be represented.

1)

2)

3)

Two matrices are equal if they have the same dimension and ____ are equal.

II. Matrix Addition and Scalar Multiplication (Pages 519–521)

What you should learn
How to add and subtract matrices and multiply matrices by scalars

To add two matrices of the same dimension, _______________ ___________________________________.

To multiply a matrix A by a scalar c, _____________________ ____.

Example 1: Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 2 & -5 \end{bmatrix}$.

Find (a) $A + B$ and (b) $-2B$

Let A, B, and C be $m \times n$ matrices and let c and d be scalars. Give an example of each of the following properties of matrix addition and scalar multiplication:

1) Commutative Property of Matrix Addition: ___________________________

2) Associative Property of Matrix Addition: ___________________________

3) Associative Property of Scalar Multiplication: _________________________

4) Scalar Identity: __________________________________

5) Additive Identity: ____________________________________

6) Distributive Property (two forms): ___________________________________

____

III. Matrix Multiplication (Pages 522–524)

What you should learn
How to multiply two matrices

The definition of matrix multiplication indicates a row-by-column multiplication, where the entry in the ith row and jth column of the product AB is obtained by ____________________

____

____.

Example 2: If A is a 3×5 matrix and B is a 6×3 matrix, find the dimension, if possible, of the product (a) AB, and (b) BA.

Example 3: Find the product AB, if

$$A = \begin{bmatrix} 2 & -1 & 7 \\ 0 & 6 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

List four properties of Matrix Multiplication:

If A is an $n \times n$ matrix, the identity matrix has the property that ______________ and ______________.

IV. Applications of Matrix Operations (Pages 525–526)

What you should learn
How to use matrix operations to model and solve real-life problems

Matrix multiplication can be used to represent a system of linear equations. The system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

can be written as the matrix equation ____________________, where A is the coefficient matrix of the system and X and B are column matrices.

Example 4: Consider the following system of linear equations.

$$\begin{cases} 2x_1 - x_2 + 3x_3 = -11 \\ x_1 - 3x_3 = -1 \\ -x_1 + 4x_2 + 2x_3 = 2 \end{cases}$$

Write this system as a matrix equation $AX = B$, and then use Gauss-Jordan elimination on the augmented matrix $[A : B]$ to solve for the matrix X.

Additional notes

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Section 7.6 The Inverse of a Square Matrix

Objective: In this lesson you learned how to find inverses of matrices and how to use inverse matrices to solve systems of linear equations.

Important Vocabulary	Define each term or concept.
Inverse of a matrix	

I. The Inverse of a Matrix (Page 532)

What you should learn
How to verify that two matrices are inverses of each other

To verify that a matrix B is the inverse of the matrix A, ________ __________________________________.

II. Finding Inverse Matrices (Pages 533–535)

What you should learn
How to use Gauss-Jordan elimination to find inverses of matrices

If a matrix A has an inverse, A is called _________________ or **nonsingular.** Otherwise, A is called ________________.

A _________________ matrix cannot have an inverse. Not all square matrices have inverses. However, if a matrix does have an inverse, that inverse is _______________.

Describe how to find the inverse of a square matrix A of dimension $n \times n$.

Example 1: Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 6 \end{bmatrix}$.

III. The Inverse of a 2 × 2 Matrix (Page 536)

What you should learn
How to use a formula to find inverses of 2 × 2 matrices

If A is a 2 × 2 matrix given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is invertible if and only if ____________________. Moreover, if this condition is true, the inverse of A is given by:

$$A^{-1} = \frac{}{} \begin{bmatrix} & \\ & \end{bmatrix}$$

The denominator is called the ______________________ of the 2 × 2 matrix A.

Example 2: Find the inverse of the matrix $B = \begin{bmatrix} 3 & 9 \\ -2 & -7 \end{bmatrix}$.

IV. Systems of Linear Equations (Page 537)

What you should learn
How to use inverse matrices to solve systems of linear equations

If A is an invertible matrix, the system of linear equations represented by $AX = B$ has a unique solution given by __________________.

Example 3: Use an inverse matrix to solve (if possible) the system of linear equations:
$$\begin{cases} 12x + 8y = 416 \\ 3x + 5y = 152 \end{cases}$$

Homework Assignment

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Exercises

Name ____ Date ____________

Section 7.7 The Determinant of a Square Matrix

Objective: In this lesson you learned how to find determinants of square matrices.

Important Vocabulary	Define each term or concept.
Determinant	
Minors	
Cofactors	

I. The Determinant of a 2 × 2 Matrix (Pages 541–542)

What you should learn
How to find the determinants of 2 × 2 matrices

The **determinant** of the 2 × 2 matrix $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ is given by

$$\det(A) = |A| = \begin{vmatrix} & \\ & \end{vmatrix} = \underline{\qquad\qquad\qquad}$$

The determinant of a matrix of dimension 1 × 1 is defined as

____.

Example 1: Find the determinant of the matrix $A = \begin{bmatrix} -4 & 3 \\ 1 & -2 \end{bmatrix}$.

II. Minors and Cofactors (Page 543)

What you should learn
How to find minors and cofactors of square matrices

Complete the sign patterns for cofactors of a 3 × 3 matrix, a 4 × 4 matrix, and a 5 × 5 matrix:

Sign Pattern for Cofactors

3 × 3 matrix	4 × 4 matrix	5 × 5 matrix
$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$	$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$	$\begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$

Example 2: Use the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ to find:

(a) the minor M_{13}, and (b) the cofactor C_{21}.

III. The Determinant of a Square Matrix (Page 544)

What you should learn
How to find the determinants of square matrices

Applying the definition of the determinant of a square matrix to find a determinant is called ______________________________.

Example 3: Find the determinant of the matrix:

$$A = \begin{bmatrix} -1 & 0 & 4 \\ 3 & -2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Example 4: Describe a strategy for finding the determinant of the following matrix, and then find the determinant of the matrix.

$$B = \begin{bmatrix} -2 & 4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 3 & 1 & -4 & -1 \\ -5 & 0 & -2 & 3 \end{bmatrix}$$

Homework Assignment

Page(s)

Exercises

Name ____ Date ____________

Section 7.8 Applications of Matrices and Determinants

Objective: In this lesson you learned how to use Cramer's Rule to solve systems of linear equations.

Important Vocabulary	Define each term or concept.
Uncoded row matrices	
Coded row matrices	

I. Area of a Triangle (Page 548)

> ***What you should learn***
> How to use determinants to find areas of triangles

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\text{Area} = \pm\frac{1}{2}\left| \quad \right|$$

where the symbol ± indicates that the appropriate sign should be chosen to yield a positive area.

Example 1: Find the area of a triangle whose vertices are $(-3, 1)$, $(2, 4)$, and $(5, -3)$.

II. Collinear Points (Page 549)

> ***What you should learn***
> How to use determinants to decide whether points are collinear

Collinear points are ______________________________.

Three points (x_1, y_1), (x_2, y_2), and (x_3, y_3) are collinear if and only if

$$\left| \quad \right| = 0.$$

Example 2: Determine whether the points $(-2, 4)$, $(0, 3)$, and $(8, -1)$ are collinear.

III. Cramer's Rule (Pages 550–552)

> ***What you should learn***
> How to use Cramer's Rule to solve systems of linear equations

Cramer's Rule states that if a system of n linear equations in n variables has a coefficient matrix A with a nonzero determinant $|A|$, the solution of the system is

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \ldots, x_n = \frac{|A_n|}{|A|}$$

where the ith column of A_i is ____________________________
____.

If the determinant of the coefficient matrix is _______________, the system has either no solution or ____________________
______________.

Example 3: Use Cramer's Rule to solve the system of linear equations.

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$$

IV. Cryptography (Pages 553–555)

> ***What you should learn***
> How to use matrices to encode and decode messages

A cryptogram is ____
_____________________________.

Describe how to use matrix multiplication to encode and decode messages.

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Chapter 8 Sequences, Series, and Probability

Section 8.1 Sequences and Series

Objective: In this lesson you learned how to use sequence, factorial, and summation notation to write the terms and sums of sequences.

> ***What you should learn***
> How to use sequence notation to write the terms of sequences

I. Sequences (Pages 570–572)

An **infinite sequence** is ______________________________
______________________________.

The function values $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ are the __________ of an infinite sequence.

A **finite sequence** is ______________________________
______________________________.

To find the first three terms of a sequence, given an expression for its nth term, ______________________________
____
______________________.

To define a sequence **recursively,** you need to be given _______
______________________________. All other terms of the sequence are then defined using ______________________.

Example 1: Find the first five terms of the sequence given by $a_n = 5 + 2n(-1)^n$.

> ***What you should learn***
> How to use factorial notation

II. Factorial Notation (Page 573)

If n is a positive integer, ***n* factorial** is defined by
____

Zero factorial is defined as ______________________.

Example 2: Evaluate the factorial expression $\dfrac{n!}{(n+1)!}$.

III. Summation Notation (Page 574)

> ***What you should learn***
> How to use summation notation to write sums

The sum of the first n terms of a sequence is represented by the **summation or sigma notation**,

$$\sum_{i=1}^{n} a_i = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

where i is called the ____________________________, n is the __________________________________, and 1 is the ___________ _________________________.

Example 3: Find the following sum: $\sum_{i=2}^{7}(2+3i)$.

IV. Series (Page 575)

> ***What you should learn***
> How to find sums of infinite series

The sum of the terms of a finite or infinite sequence is called a ____________________.

Consider the infinite sequence $a_1, a_2, a_3, \ldots, a_i, \ldots$. The sum of the first n terms of the sequence is called a(n) ______________________ or the ________________________ of the sequence and is denoted by $a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$. The sum of all terms of the infinite sequence is called a(n) _________________________ and is denoted by $a_1 + a_2 + a_3 + \cdots + a_i + \cdots = \sum_{i=1}^{\infty} a_i$.

> **Homework Assignment**
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Name ____ Date ____________

Section 8.2 Arithmetic Sequences and Partial Sums

Objective: In this lesson you learned how to recognize, write, and use arithmetic sequences.

Important Vocabulary	Define each term or concept.
Arithmetic sequence	

I. Arithmetic Sequences (Pages 581–583)

What you should learn
How to recognize, write, and find the *n*th terms of arithmetic sequences

Define the common difference of an arithmetic sequence.

Example 1: Determine whether or not the following sequence is arithmetic. If it is, find the common difference.
$7, 3, -1, -5, -9, \ldots$

The *n*th term of an arithmetic sequence has the form

________________________, where d is the common

difference between consecutive terms of the sequence, and a_1 is the first term of the sequence.

Example 2: Find a formula for the *n*th term of the arithmetic sequence whose common difference is 2 and whose first term is 7.

When you know the *n*th term of an arithmetic sequence *and* you know the common difference of the sequence, you can find the $(n + 1)$th term by using the recursion formula

______________________________.

Example 3: Find the sixth term of the arithmetic sequence that begins with 15 and 12.

II. The Sum of a Finite Arithmetic Sequence (Page 584)

> ***What you should learn***
> How to find *n*th partial sums of arithmetic sequences

The sum of a finite arithmetic sequence with n terms is given by

____ .

The sum of the first n terms of an infinite sequence is called the

____________________________________.

Example 4: Find the sum of the first 20 terms of the sequence with nth term $a_n = 28 - 5n$.

III. Applications (Page 585)

> ***What you should learn***
> How to use arithmetic sequences to model and solve real-life problems

Describe a real-life problem that could be solved by finding the sum of a finite arithmetic sequence.

Homework Assignment

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Exercises

Name ____ Date ___________

Section 8.3 Geometric Sequences and Series

Objective: In this lesson you learned how to recognize, write, and use geometric sequences.

Important Vocabulary	Define each term or concept.
Geometric sequence	
Infinite geometric series or geometric series	

I. Geometric Sequences (Pages 589–591)

What you should learn
How to recognize, write, and find the nth terms of geometric sequences

Define the common ratio of a geometric sequence.

Example 1: Determine whether or not the following sequence is geometric. If it is, find the common ratio.
60, 30, 0, – 30, – 60, . . .

The nth term of a geometric sequence has the form

__________________, where r is the common ratio of

consecutive terms of the sequence. So, every geometric sequence

can be written in the following form:

_______________________________________ .

If you know the nth term of a geometric sequence, you can find

the $(n + 1)$th term by ______________________. That is,

$a_{n+1} =$ ________________ .

Example 2: Write the first five terms of the geometric sequence whose first term is $a_1 = 5$ and whose common ratio is – 3.

Example 3: Find the eighth term of the geometric sequence that begins with 15 and 12.

II. The Sum of a Finite Geometric Sequence (Page 592)

> ***What you should learn***
> How to find *n*th partial sums of geometric sequences

The sum of the finite geometric sequence $a_1, a_1r, a_1r^2, a_1r^3, a_1r^4,$ $\ldots, a_1r^{n-1}$ with common ratio $r \neq 1$ is given by

________________________________ .

When using the formula for the sum of a geometric sequence, be careful to check that the index begins with $i = 1$. If the index begins at $i = 0$, ______________________________________

______________________________________.

Example 4: Find the sum $\sum_{i=1}^{10} 2(0.5)^i$.

III. Geometric Series (Pages 593)

> ***What you should learn***
> How to find sums of infinite geometric series

If $|r| < 1$, then the infinite geometric series $a_1 + a_1r + a_1r^2 + a_1r^3$ $+ a_1r^4 + \ldots + a_1r^{n-1} + \ldots$ has the sum ____________________ .

If $|r| \geq 1$, the series ___________________________ a sum.

Example 5: If possible, find the sum: $\sum_{i=1}^{\infty} 9(0.25)^{i-1}$.

IV. Applications (Page 594)

> ***What you should learn***
> How to use geometric sequences to model and solve real-life problems

Describe a real-life problem that could be solved by finding the sum of a finite geometric sequence.

Homework Assignment

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Exercises

Name ____ Date ____________

Section 8.4 The Binomial Theorem

Objective: In this lesson you learned how to use the Binomial Theorem and Pascal's Triangle to calculate binomial coefficients and write binomial expansions.

Important Vocabulary Define each term or concept.

Binomial coefficients

Pascal's Triangle

I. Binomial Coefficients (Pages 599–600)

What you should learn
How to use the Binomial Theorem to calculate binomial coefficients

List four general observations about the expansion of $(x + y)^n$ for various values of n.

1)

2)

3)

4)

The **Binomial Theorem** states that in the expansion of $(x + y)^n =$ $x^n + nx^{n-1}y + \ldots + {}_nC_r x^{n-r}y^r + \ldots + nxy^{n-1} + y^n$, the coefficient of $x^{n-r}y^r$ is ____________________________ .

Example 1: Find the binomial coefficient ${}_{12}C_5$.

II. Binomial Expansions (Pages 601–602)

What you should learn
How to use binomial coefficients to write binomial expansions

Writing out the coefficients for a binomial that is raised to a power is called _______________________________.

Example 2: Write the expansion of the expression $(x+2)^5$.

III. Pascal's Triangle (Page 603)

Construct rows 0 through 6 of Pascal's Triangle.

What you should learn
How to use Pascal's Triangle to calculate binomial coefficients

Additional notes

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Exercises

Name ____ Date ____________

Section 8.5 Counting Principles

Objective: In this lesson you learned how to solve counting problems using the Fundamental Counting Principle, permutations, and combinations.

Important Vocabulary Define each term or concept.

Fundamental Counting Principle

Permutation

Distinguishable permutations

I. Simple Counting Problems (Page 607)

What you should learn
How to solve simple counting problems

If two balls are randomly drawn from a bag of six balls, numbered from 1 to 6, such that it is possible to choose two 3's, the random selection occurs __________________________. If two balls are drawn from the bag at the same time, the random selection occurs ____________________________, which eliminates the possibility of choosing two 3's.

II. The Fundamental Counting Principle (Page 608)

What you should learn
How to use the Fundamental Counting Principle to solve more complicated counting problems

The Fundamental Counting Principle can be extended to three or more events. For instance, if E_1 can occur in m_1 ways, E_2 in m_2 ways, and E_3 in m_3 ways, the number of ways that three events E_1, E_2, and E_3 can occur is ________________________.

Example 1: A diner offers breakfast combination plates which can be made from a choice of one of 4 different types of breakfast meats, one of 8 different styles of eggs, and one of 5 different types of breakfast breads. How many different breakfast combination plates are possible?

III. Permutations (Pages 609–611)

> ***What you should learn***
> How to use permutations to solve counting problems

The number of different ways that n elements can be ordered is ______________________________.

A **permutation of *n* elements taken *r* at a time** is ___________
____
____
____.

The number of n elements taken r at a time is given by
_______________________________________ .

Example 2: In how many ways can a chairperson, a vice chairperson, and a recording secretary be chosen from a committee of 14 people?

Example 3: In how many distinguishable ways can the letters COMMITTEE be written?

IV. Combinations (Pages 612)

> ***What you should learn***
> How to use combinations to solve counting problems

A **combination of *n* elements taken *r* at a time** is ___________
____.

The number of combinations of n elements taken r at a time is given by ____ .

Example 4: In how many ways can a research team of 3 students be chosen from a class of 14 students?

> **Homework Assignment**
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Name ____ Date ____________

Section 8.6 Probability

Objective: In this lesson you learned how to find the probability of events.

I. The Probability of an Event (Pages 616–618)

> ***What you should learn***
> How to find probabilities of events

Any happening whose result is uncertain is called a(n) ___________________. The possible results of the experiment are ____________________, the set of all possible outcomes of the experiment is the ______________________ of the experiment, and any subcollection of a sample space is a(n) _______________.

To calculate the probability of an event, ___________________ ____.

If an event *E* has $n(E)$ equally likely outcomes and its sample space *S* has $n(S)$ equally likely outcomes, the **probability** of event *E* is given by ___________________________.

The probability of an event must be between ______ and _______.

If $P(E) = 0$, the event *E* _______________ occur, and *E* is called a(n) __________________ event. If $P(E) = 1$, the event *E* _______________ occur, and *E* is called a(n) _______________ event.

Example 1: A box contains 3 red marbles, 5 black marbles, and 2 yellow marbles. If a marble is selected at random from the box, what is the probability that it is yellow?

II. Mutually Exclusive Events (Pages 619–620)

> ***What you should learn***
> How to find probabilities of mutually exclusive events

Two events *A* and *B* (from the same sample space) are __________ ________________ when *A* and *B* have no outcomes in common.

To find the probability that one or the other of two mutually exclusive events will occur, ______________________________ ____.

If A and B are events in the same sample space, the probability of A *or* B occurring is given by $P(A \cup B) =$ ______________________.

If A and B are mutually exclusive, then $P(A \cup B) =$ ___________________.

Example 2: A box contains 3 red marbles, 5 black marbles, and 2 yellow marbles. If a marble is selected at random from the box, what is the probability that it is either red or black?

III. Independent Events (Page 621)

> ***What you should learn***
> How to find probabilities of independent events

Two events are **independent** when ___________________________ ____.

If A and B are **independent events,** the probability that both A and B will occur is $P(A \text{ and } B) =$ ___________________________.

That is, to find the probability that two independent events will occur, ____.

Example 3: A box contains 3 red marbles, 5 black marbles, and 2 yellow marbles. If two marbles are randomly selected with replacement, what is the probability that both marbles are yellow?

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Chapter 9 Topics in Analytic Geometry

Section 9.1 Circles and Parabolas

Objective: In this lesson you learned how to recognize conics, write equations of circles in standard form, write equations of parabolas in standard form, and use the reflective property of parabolas to solve problems.

Important Vocabulary Define each term or concept.

Directrix

Focus

Tangent

I. Conics (Page 636)

What you should learn
How to recognize a conic as the intersection of a plane and a double-napped cone

A **conic section,** or **conic,** is ____________________________
____.

Name the four basic conic sections: __________________
____.

In the formation of the four basic conics, the intersecting plane does not pass through the vertex of the cone. When the plane does pass through the vertex, the resulting figure is a(n) _________________________________, such as
____
____

II. Circles (Pages 637–638)

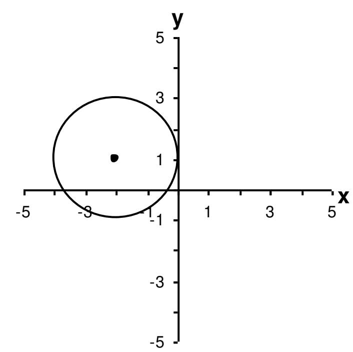
What you should learn
How to write equations of circles in standard form

A **circle** is the set of all points (x, y) in a plane that are _________________________ from a fixed point (h, k), called the ____________________ of the circle. The distance r between the center and any point (x, y) on the circle is the _______________.

The **standard form of the equation of a circle** with center (h, k) and radius r is ________________________ .

The standard form of the equation of a circle with radius r and whose center is the origin is ________________________ .

Example 1: The point (0, 1) is on a circle whose center is (−2, 1), as shown in the figure. Write the standard form of the equation of the circle.



III. Parabolas (Pages 639–640)

> ***What you should learn***
> How to write equations of parabolas in standard form

A **parabola** is ____
____
____.

The midpoint between the focus and the directrix is the ________________ of a parabola. The line passing through the focus and the vertex is the ________________ of the parabola.

The standard form of the equation of a parabola with a vertical axis having a vertex at (h, k) and directrix $y = k - p$ is

____

The standard form of the equation of a parabola with a horizontal axis having a vertex at (h, k) and directrix $x = h - p$ is

____

The focus lies on the axis p units (directed distance) from the vertex. If the vertex is at the origin (0, 0), the equation takes one of the following forms:

____

Example 2: Find the standard form of the equation of the parabola with vertex at the origin and focus (1, 0).

IV. Reflective Property of Parabolas (Pages 641–642)

> ***What you should learn***
> How to use the reflective property of parabolas to solve real-life problems

Describe a real-life situation in which parabolas are used.

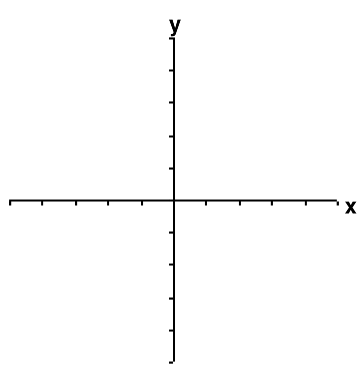
A **focal chord** is ____
____.

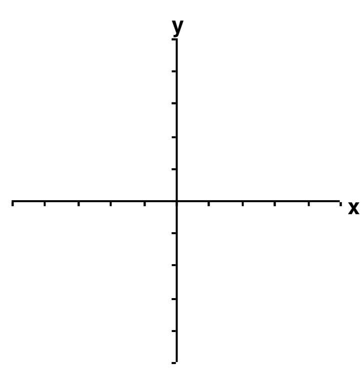
The specific focal chord perpendicular to the axis of a parabola is called the ______________________________.

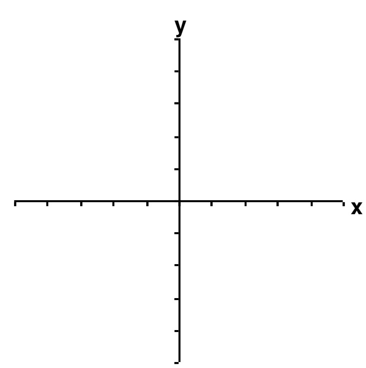
The reflective property of a parabola states that the tangent line to a parabola at a point *P* makes equal angles with the following two lines:

1)

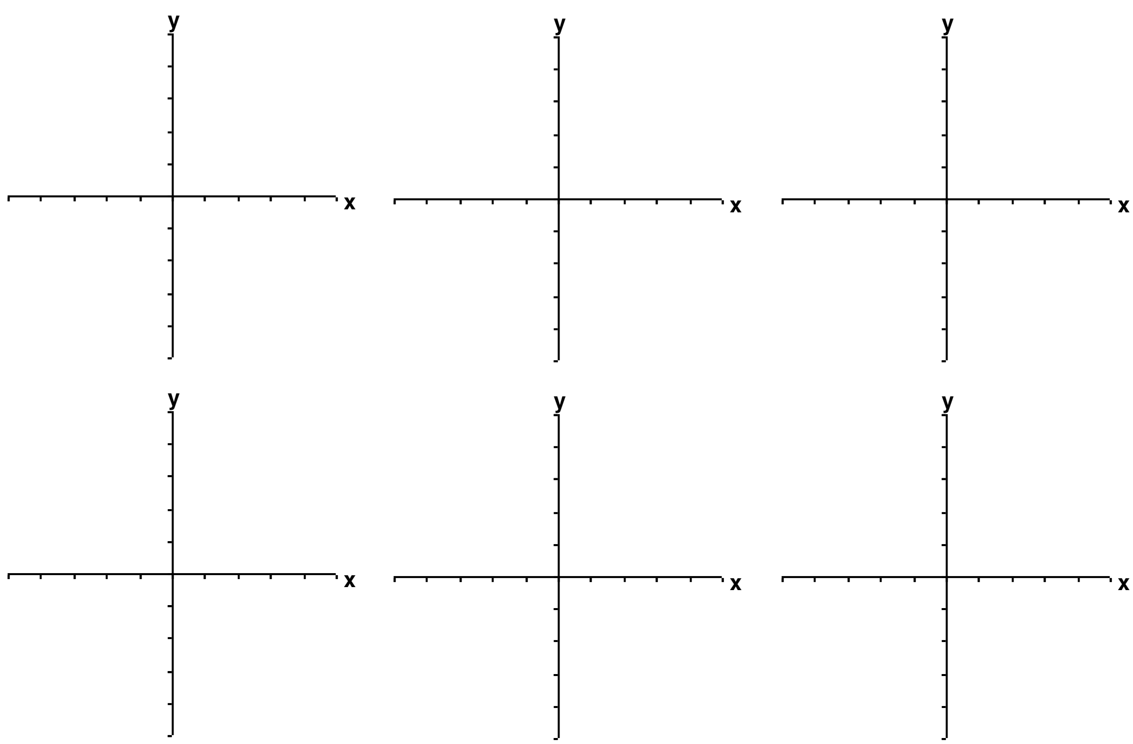
2)







Additional notes



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Section 9.2 Ellipses

Objective: In this lesson you learned how to write the standard form of the equation of an ellipse, and analyze and sketch the graphs of ellipses.

Important Vocabulary	Define each term or concept.
Foci	
Vertices	
Major axis	
Center	
Minor axis	

I. Introduction (Pages 647–650)

What you should learn
How to write equations of ellipses in standard form

An **ellipse** is ____
____
______________________________________.

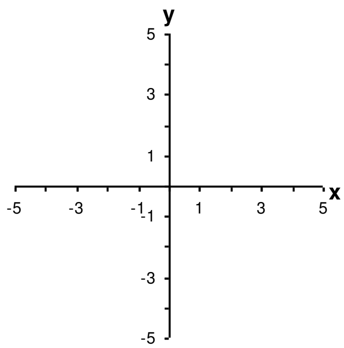
The standard form of the equation of an ellipse with center (h, k) and a horizontal major axis of length $2a$ and a minor axis of length $2b$, where $0 < b < a$, is: ____________________________________

The standard form of the equation of an ellipse with center (h, k) and a vertical major axis of length $2a$ and a minor axis of length $2b$, where $0 < b < a$, is: ____________________________________

In both cases, the foci lie on the major axis, c units from the center, with $c^2 =$ ______________ .

If the center is at the origin $(0, 0)$, the equation takes one of the following forms: ____________________________ or
____________________________.

Example 1: Sketch the ellipse given by $4x^2 + 25y^2 = 100$.



II. Applications (Page 651)

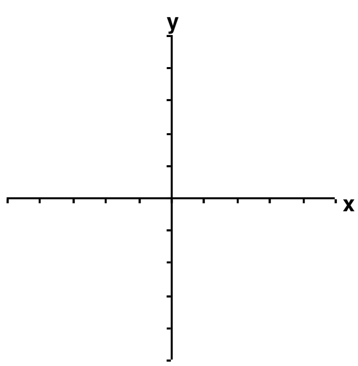
> ***What you should learn***
> How to use properties of ellipses to model and solve real-life problems

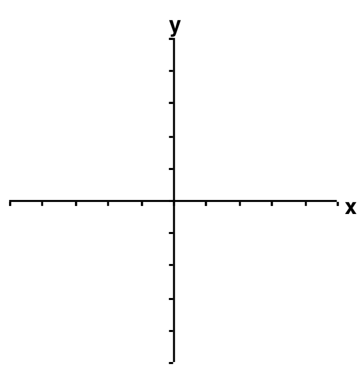
Describe a real-life application in which parabolas are used.

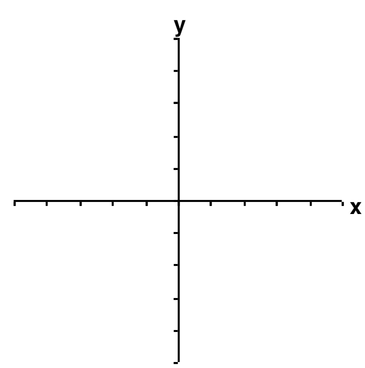
III. Eccentricity (Page 652)

> ***What you should learn***
> How to find eccentricities of ellipses

____________________ measures the ovalness of an ellipse. It is given by the ratio e = _____________. For every ellipse, the value of e lies between ________ and ________. For an elongated ellipse, the value of e is close to ____________.







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Section 9.3 Hyperbolas and Rotation of Conics

Objective: In this lesson you learned how to write the standard form of the equation of a hyperbola, analyze and sketch the graphs of hyperbolas, and rotate axes.

Important Vocabulary	Define each term or concept.
Branches	
Transverse axis	
Conjugate axis	

I. Introduction (Pages 656–657)

What you should learn
How to write equations of hyperbolas in standard form

A **hyperbola** is ____
____
____.

The line through a hyperbola's two foci intersects the hyperbola at two points called ____________________________.

The midpoint of a hyperbola's transverse axis is the __________________ of the hyperbola.

The standard form of the equation of a hyperbola centered at (h, k) and having a horizontal transverse axis is

____

The standard form of the equation of a hyperbola centered at (h, k) and having a vertical transverse axis is

____

In each case, the vertices and foci are, respectively, a and c units from the center. Moreover, a, b, and c are related by the equation ____________________.

If the center of the hyperbola is at the origin $(0, 0)$, the equation takes one of the following forms: ________________________ or ________________________.

II. Asymptotes of a Hyperbola (Pages 658–660)

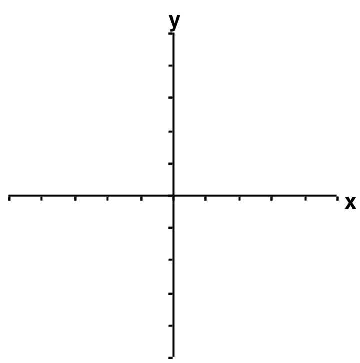
> ***What you should learn***
> How to find asymptotes of and graph hyperbolas

The **asymptotes** of a hyperbola with a horizontal transverse axis are ____________________.

The **asymptotes** of a hyperbola with a vertical transverse axis are _____________________.

The asymptotes pass through the corners of a rectangle of dimensions _____________ by ______________, with its center at ___________________.

Example 1: Sketch the graph of the hyperbola given by $y^2 - 9x^2 = 9$.



The **eccentricity** of a hyperbola is e = _______________, where the values of e are ______________________.

III. Applications of Hyperbolas (Page 661)

> ***What you should learn***
> How to use properties of hyperbolas to solve real-life problems

Describe a real-life application in which hyperbolas occur or are used.

IV. General Equations of Conics (Page 662)

> ***What you should learn***
> How to classify conics from their general equations

The graph of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is one of the following:

1) Circle if ________________
2) Parabola if ________________
3) Ellipse if ________________
4) Hyperbola if ________________

Example 2: Classify the equation $9x^2 + y^2 - 18x - 4y + 4 = 0$ as a circle, a parabola, an ellipse, or a hyperbola.

V. Rotation (Pages 663–664)

> ***What you should learn***
> How to rotate the coordinate axes to eliminate the *xy*-term in equations of conics

The general equation of a conic whose axes are rotated so that they are not parallel to either the *x*-axis or the *y*-axis contains a(n) _________________.

To eliminate this term, you can use a procedure called _________ _____________, whose objective is to rotate the *x*- and *y*-axes until they are parallel to the axes of the conic.

The general second-degree equation

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ can be rewritten as

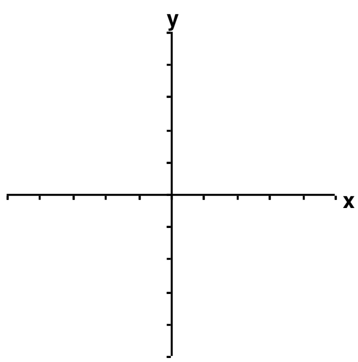
$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$ by rotating the

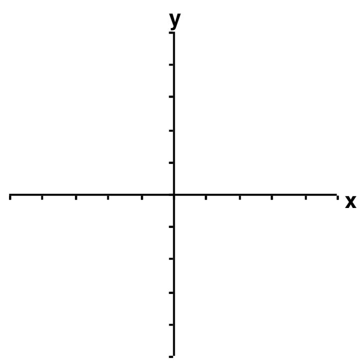
coordinate axes through an angle θ, where

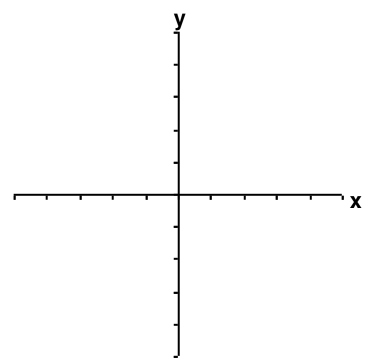
$\cot 2\theta =$ _______________________.

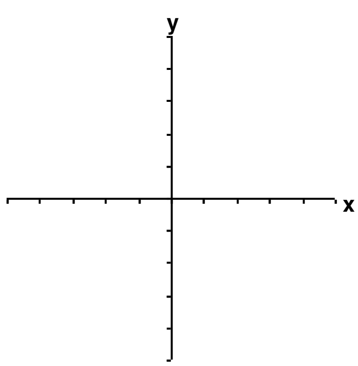
The coefficients of the new equation are obtained by making the substitutions $x =$ ________________________________ and

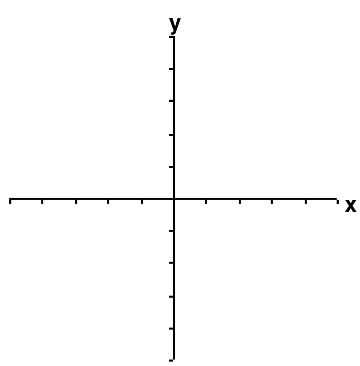
$y =$ ______________________________.

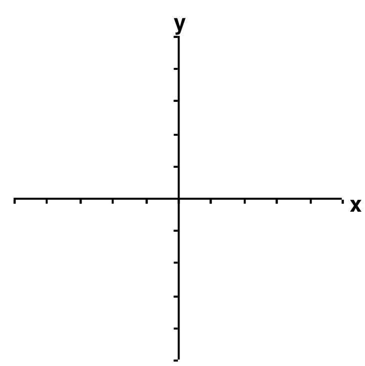












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Section 9.4 Parametric Equations

Objective: In this lesson you learned how to evaluate sets of parametric equations for given values of the parameter and graph curves that are represented by sets of parametric equations and how to rewrite sets of parametric equations as single rectangular equations and find sets of parametric equations for graphs.

Important Vocabulary Define each term or concept.

Parameter

I. Plane Curves (Page 669)

What you should learn
How to evaluate sets of parametric equations for given values of the parameter

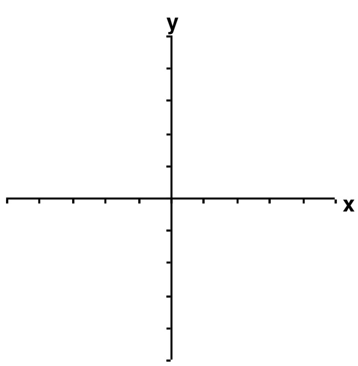
If f and g are continuous functions of t on an interval I, the set of ordered pairs $(f(t), g(t))$ is a(n) ____________________ C. The equations given by $x = f(t)$ and $y = g(t)$ are ________________ ________________ for C, and t is the __________________.

II. Graphs of Plane Curves (Pages 670–671)

What you should learn
How to graph curves that are represented by sets of parametric equations

One way to sketch a curve represented by a pair of parametric equations is to plot points in the _________________. Each set of coordinates (x, y) is determined from a value chosen for the ____________________. By plotting the resulting points in the order of increasing values of t, you trace the curve in a specific direction, called the ___________________ of the curve.

Example 1: Sketch the curve described by the parametric equations $x = t - 3$ and $y = t^2 + 1$, $-1 \le t \le 3$.



Describe how to display a curve represented by a pair of parametric equations using a graphing utility.

III. Eliminating the Parameter (Pages 672)

Eliminating the parameter is the process of ________________
____
_______________________________________.

Describe the process used to eliminate the parameter from a set of parametric equations.

When converting equations from parametric to rectangular form, it may be necessary to alter ___________________________
____
_______________________________________.

What you should learn
How to rewrite sets of parametric equations as single rectangular equations by eliminating the parameter

IV. Finding Parametric Equations for a Graph (Page 673)

Describe how to find a set of parametric equations for a given graph.

What you should learn
How to find sets of parametric equations for graphs

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Section 9.5 Polar Coordinates

Objective: In this lesson you learned how to plot points in the polar coordinate system and convert equations from rectangular to polar form and vice versa.

I. Introduction (Pages 677–678)

> ***What you should learn***
> How to plot points and find multiple representations of points in the polar coordinate system

To form the **polar coordinate system** in the plane, fix a point O, called the ______________ or ________________, and construct from O an initial ray called the ___________________. Then each point P in the plane can be assigned _________________ ________ as follows:

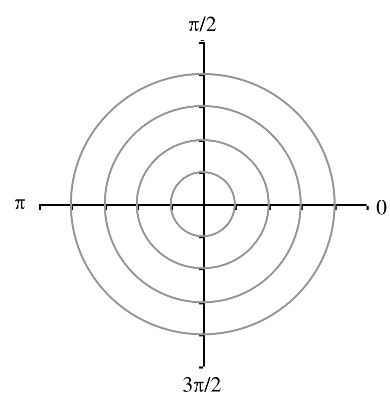
1) $r =$ ______________________________________

2) $\theta =$ ____

____

In the polar coordinate system, points do not have a unique representation. For instance, the point (r, θ) can be represented as ____________________ or __________________________, where n is any integer. Moreover, the pole is represented by $(0, \theta)$, where θ is _____________________.

Example 1: Plot the point $(r, \theta) = (-2, 11\pi/4)$ on the polar coordinate system.



Example 2: Find another polar representation of the point $(4, \pi/6)$.

II. Coordinate Conversion (Page 679)

> ***What you should learn***
> How to convert points from rectangular to polar form and vice versa

The polar coordinates (r, θ) are related to the rectangular coordinates (x, y) as follows:

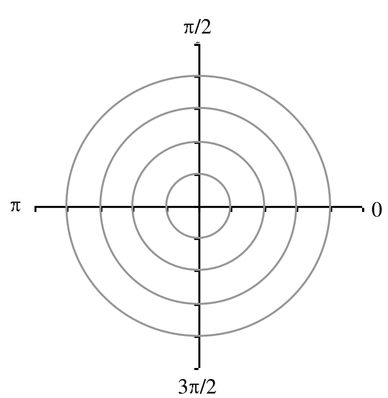
Example 3: Convert the polar coordinates $(3, 3\pi/2)$ to rectangular coordinates.

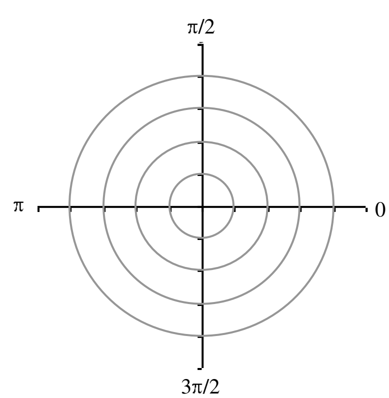
III. Equation Conversion (Page 680)

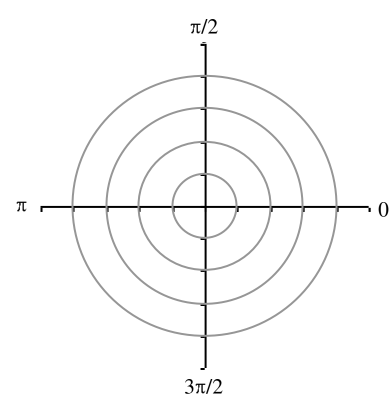
> ***What you should learn***
> How to convert equations from rectangular to polar form and vice versa

To convert a rectangular equation to polar form, ____________ ____.

Example 4: Find the rectangular equation corresponding to the polar equation $r = \dfrac{-5}{\sin\theta}$.







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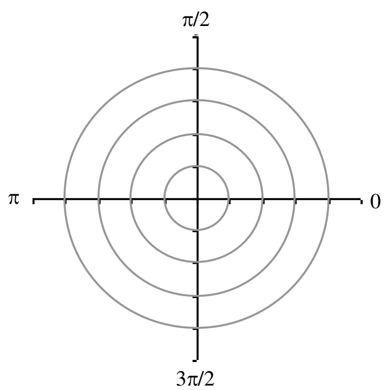
Section 9.6 Graphs of Polar Equations

Objective: In this lesson you learned how to graph polar equations.

> ***What you should learn***
> How to graph polar equations by point plotting

I. Introduction (Page 683)

Example 1: Use point plotting to sketch the graph of the polar equation $r = 3\cos\theta$.



The graph of the polar equation $r = f(\theta)$ can be rewritten in parametric form, using t as a parameter, as follows:

____

> ***What you should learn***
> How to use symmetry and zeros as sketching aids

II. Symmetry and Zeros (Pages 684–686)

The graph of a polar equation is symmetric with respect to the following if the given substitution yields an equivalent equation.

Substitution

1) The line $\theta = \pi/2$:

2) The polar axis:

3) The pole:

Example 2: Describe the symmetry of the polar equation $r = 2(1 - \sin\theta)$.

An additional aid to sketching graphs of polar equations is

____.

Example 3: Describe the zeros of the polar equation
$r = 5\cos 2\theta$

IV. Special Polar Graphs (Pages 687–688)

> ***What you should learn***
> How to recognize special polar graphs

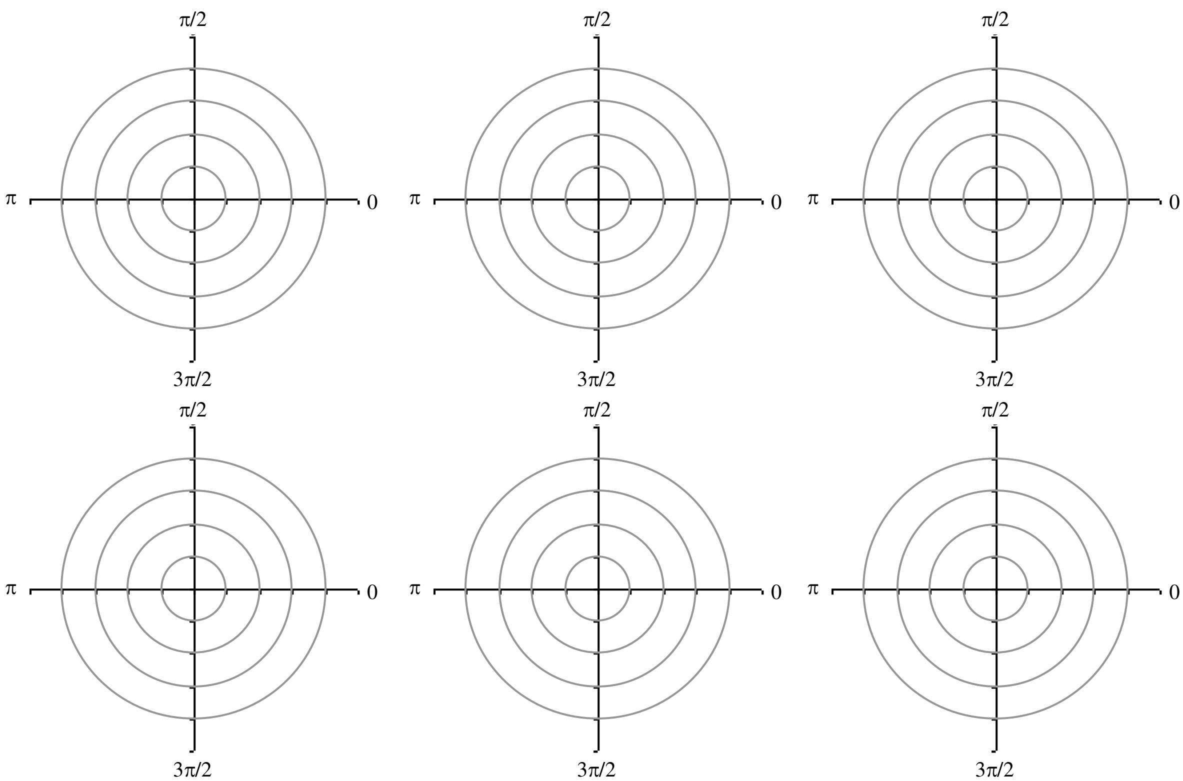
List the general equations that yield each of the following types of special polar graphs:

Limaçons:

Rose curves:

Circles:

Lemniscates:



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Section 9.7 Polar Equations of Conics

Objective: In this lesson you learned how to write conics in terms of eccentricity and to write equations of conics in polar form.

I. Alternative Definition of Conics and Polar Equations (Pages 691–693)

What you should learn
How to define conics in terms of eccentricities and write and graph equations of conics in polar form

The locus of a point in the plane that moves so that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a _______________. The constant ratio is the _______________________ of the conic and is denoted by e. Moreover, the conic is an ellipse if ____________, a parabola if ____________, and a hyperbola if _____________.

For each type of conic, the focus is at the ______________.

The graph of the polar equation __________________________ is a conic with a vertical directrix to the right of the pole, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

The graph of the polar equation __________________________ is a conic with a vertical directrix to the left of the pole, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

The graph of the polar equation __________________________ is a conic with a horizontal directrix above the pole, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

The graph of the polar equation __________________________ is a conic with a horizontal directrix below the pole, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

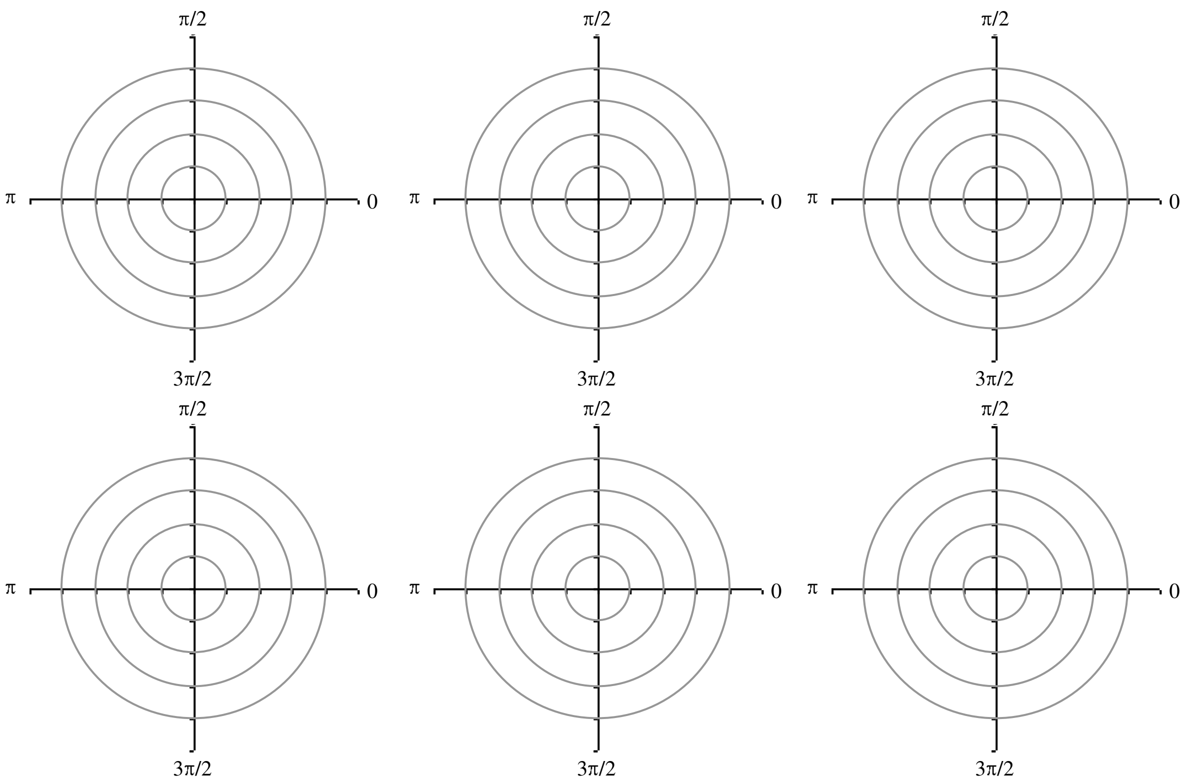
Example 1: Identify the type of conic from the polar equation

$r = \dfrac{36}{10 + 12\sin\theta}$, and describe its orientation.

II. Applications (Page 694)

> ***What you should learn***
> How to use equations of conics in polar form to model real-life problems

Describe a real-life application of polar equations of conics.



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Chapter 10 Analytic Geometry in Three Dimensions

Section 10.1 The Three-Dimensional Coordinate System

Objective: In this lesson you learned how to plot points, find distances between points, and find midpoints of line segments connecting points in space and how to write equations of spheres and graph traces of surfaces in space.

Important Vocabulary	Define each term or concept.
Solid analytic geometry	
Sphere	
Surface in space	
Trace	

I. The Three-Dimensional Coordinate System (Page 712)

What you should learn
How to plot points in the three-dimensional coordinate system

A **three-dimensional coordinate system** is constructed by

____

____.

Taken as pairs, the axes determine three **coordinate planes:** the ______________, the ______________, and the ______________.

These three coordinate planes separate the three-dimensional coordinate system into eight ________________. The first octant is the one in which ____________________________________

___________________________________.

In the three-dimensional system, a point P in space is determined by an ordered triple (x, y, z), where x, y, and z are as follows.

$x =$ ____

$y =$ ____

$z =$ ____

II. The Distance and Midpoint Formulas (Page 713)

> ***What you should learn***
> How to find distances between points in space and find midpoints of line segments joining points in space

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) given by the **Distance Formula in Space** is

$$d = \sqrt{\quad}$$

The midpoint of the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) given by the **Midpoint Formula in Space** is

$$\left(\qquad \right)$$

Example 1: For the points $(2, 0, -4)$ and $(-1, 4, 6)$, find
(a) the distance between the two points, and
(b) the midpoint of the line segment joining them.

III. The Equation of a Sphere (Pages 714–715)

> ***What you should learn***
> How to write equations of spheres in standard form and find traces of surfaces in space

The standard equation of a sphere whose center is (h, k, j) and whose radius is r is ____________________________________.

Example 2: Find the center and radius of the sphere whose equation is $x^2 + y^2 + z^2 - 4x + 2y + 8z + 17 = 0$.

To find the yz-trace of a surface, _________________________
____
____
____
____.

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Section 10.2 Vectors in Space

Objective: In this lesson you learned how to represent vectors and find dot products of and angles between vectors in space.

Important Vocabulary Define each term or concept.

Standard unit vector notation in space

Angle between two nonzero vectors in space

Parallel vectors in space

I. Vectors in Space (Pages 719–720)

What you should learn
How to find the component forms of, the unit vectors in the same direction of, the magnitudes of, the dot products of, and the angles between vectors in space

In space, vectors are denoted by ordered triples of the form

______________________ .

The **zero vector in space** is denoted by ______________.

If **v** is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, describe how the **component form** of **v** is produced.

Two vectors are **equal** if and only if ________________

___________________________________.

The **magnitude**(or length) of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ is:

$\| \mathbf{u} \| = \sqrt{\qquad\qquad\qquad\qquad}$

A unit vector **u** in the direction of **v** is ________________.

The **sum** of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$\mathbf{u} + \mathbf{v} =$ ______________________ .

The **scalar multiple** of the real number c and $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ is

$c\mathbf{u}$ = ______________________ .

The **dot product** of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$\mathbf{u} \bullet \mathbf{v}$ = ___________________________

If θ is the angle between two nonzero vectors **u** and **v**, then

$\cos\theta$ = __________________________________.

If the dot product of two nonzero vectors is zero, the angle between the vectors is ____________. Such vectors are called ___________________.

Example 1: Find the dot product of the vectors $\langle -1, 4, -2 \rangle$ and $\langle 0, -1, 5 \rangle$.

II. Parallel Vectors (Pages 721–722)

> ***What you should learn***
> How to determine whether vectors in space are parallel or orthogonal

Example 2: Determine whether the vectors $\langle 6, 1, -3 \rangle$ and $\langle -2, -1/3, 1 \rangle$ are parallel.

To use vectors to determine whether three points P, Q, and R in space are collinear, __________________________________ _______________________________________.

III. Application (Page 723)

> ***What you should learn***
> How to use vectors in space to solve real-life problems

Describe a real-life application of vectors in space.

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Section 10.3 The Cross Product of Two Vectors

Objective: In this lesson you learned how to find cross products of vectors in space, use geometric properties of the cross product, and use triple scalar products to find volumes of parallelepipeds.

I. The Cross Product (Pages 726–727)

> ***What you should learn***
> How to find cross products of vectors in space

A vector in space that is orthogonal to two given vectors is called their ______________________.

Let $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ be two vectors in space. The **cross product** of **u** and **v** is the vector

$\mathbf{u} \times \mathbf{v} =$ ____

Describe a convenient way to remember the formula for the cross product.

Example 1: Given $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find the cross product $\mathbf{u} \times \mathbf{v}$.

Let **u**, **v**, and **w** be vectors in space and let c be a scalar. Complete the following properties of the cross product:

1. $\mathbf{u} \times \mathbf{v} =$ ___________________
2. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) =$ _________________________
3. $c(\mathbf{u} \times \mathbf{v}) =$ ____________________________
4. $\mathbf{u} \times \mathbf{0} =$ ___________________
5. $\mathbf{u} \times \mathbf{u} =$ ____________
6. $\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) =$ _____________________

II. Geometric Properties of the Cross Product
(Pages 728–729)

What you should learn
How to use geometric properties of cross products of vectors in space

Complete the following geometric properties of the cross product, given **u** and **v** are nonzero vectors in space and θ is the angle between **u** and **v**.

1. $\mathbf{u} \times \mathbf{v}$ is orthogonal to ____________________.

2. $\| \mathbf{u} \times \mathbf{v} \| =$ ____________________________.

3. $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if _______________________________.

4. $\| \mathbf{u} \times \mathbf{v} \| =$ area of the parallelogram having _______________ ____________________.

III. The Triple Scalar Product (Page 730)

What you should learn
How to use triple scalar products to find volumes of parallelepipeds

For vectors **u**, **v**, and **w** in space, the dot product of **u** and $\mathbf{v} \times \mathbf{w}$ is called the ____________________________ of **u**, **v**, and **w**, and is found as

$$\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) = \left| \quad\quad\quad \right|$$

The volume V of a parallelepiped with vectors **u**, **v**, and **w** as adjacent edges is _________________________.

Example 2: Find the volume of the parallelepiped having $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, and $\mathbf{w} = 4\mathbf{i} - 3\mathbf{k}$ as adjacent edges.

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Section 10.4 Lines and Planes in Space

Objective: In this lesson you learned how to find parametric and symmetric equations of lines in space and find distances between points and planes in space.

I. Lines in Space (Pages 733–734)

What you should learn
How to find parametric and symmetric equations of lines in space

For the line L through the point $P = (x_1, y_1, z_1)$ and parallel to the vector $\mathbf{v} = \langle a, b, c\rangle$, the vector v is the ______________________ for the line L, and the values a, b, and c are the ______________ ______________.

One way of describing the line L is ______________________ ____ ____.

A line L parallel to the nonzero vector $\mathbf{v} = \langle a, b, c\rangle$ and passing through the point $P = (x_1, y_1, z_1)$ is represented by the following parametric equations, where t is the parameter:

____

If the direction numbers a, b, and c are all nonzero, you can eliminate the parameter t to obtain the **symmetric equations** of a line:

____

II. Planes in Space (Pages 735–737)

What you should learn
How to find equations of planes in space

The plane containing the point (x_1, y_1, z_1) and having nonzero normal vector $\mathbf{n} = \langle a, b, c\rangle$ can be represented by the **standard form of the equation of a plane,** which is

____

By regrouping terms, you obtain the **general form of the equation of a plane** in space:

____

To find a normal vector to a plane given the general form of the equation of the plane, _________________________________ ____.

Two distinct planes in three-space either are ________________
or ___________________________.

If two distinct planes intersect, the **angle θ between the two planes** is equal to the angle between vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ normal to the two intersecting planes, and is given by

____

Consequently, two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are

1. _____________________ if $\mathbf{n}_1 \bullet \mathbf{n}_2 = 0$.
2. __________________ if $\mathbf{n}_1$ is a scalar multiple of $\mathbf{n}_2$.

III. Sketching Planes in Space (Page 738)

What you should learn
How to sketch planes in space

If a plane in space intersects one of the coordinate planes, the line of intersection is called the ______________ of the given plane in the coordinate plane.

To sketch a plane in space, _____________________________
____
____.

The plane with equation $3y - 2z + 1 = 0$ is parallel to
_____________________.

IV. Distance Between a Point and a Plane (Page 739)

What you should learn
How to find distances between points and planes in space

The **distance between a plane and a point** Q (not in the plane) is

____

where P is a point in the plane and $\mathbf{n}$ is normal to the plane.

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Chapter 11 Limits and an Introduction to Calculus

Section 11.1 Introduction to Limits

Objective: In this lesson you learned how to estimate limits and use properties and operations of limits.

I. The Limit Concept and Definition of Limit (Pages 750–752)

What you should learn
How to understand the limit concept and use the definition of a limit to estimate limits

Define **limit.**

Describe how to estimate the limit $\lim\limits_{x \to -2} \dfrac{x^2 + 4x + 4}{x + 2}$ numerically.

The existence or nonexistence of $f(x)$ when $x = c$ has no bearing on the existence of ______________________________.

II. Limits That Fail to Exist (Pages 753–754)

What you should learn
How to determine whether limits of functions exist

The limit of $f(x)$ as $x \to c$ does not exist under any of the following conditions.

1.

2.

3.

Give an example of a limit that does not exist.

III. Properties of Limits and Direct Substitution
(Pages 755–756)

What you should learn
How to use properties of limits and direct substitution to evaluate limits

Let b and c be real numbers and let n be a positive integer. Complete each of the following properties of limits.

1. $\lim_{x \to c} b =$ ____________
2. $\lim_{x \to c} x =$ ____________
3. $\lim_{x \to c} x^n =$ ____________
4. $\lim_{x \to c} \sqrt[n]{x} =$ ________________________________

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

Complete each of the following statements about operations with limits.

1. Scalar multiple: $\lim_{x \to c} [b\,f(x)] =$ ______________
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] =$ _______________
3. Product: $\lim_{x \to c} [f(x) \cdot g(x)] =$ _____________
4. Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} =$ ________________________
5. Power: $\lim_{x \to c} [f(x)]^n =$ _____________

Example 1: Find the limit: $\lim_{x \to 4} 3x^2$.

If p is a polynomial function and c is a real number, then

$\lim_{x \to c} p(x) =$ _______________.

If r is a rational function given by $r(x) = p(x)/q(x)$, and c is a real number such that $q(c) \neq 0$, then

$\lim_{x \to c} r(x) =$ ___________________________.

Example 2: Find the limit: $\lim_{x \to 2} \dfrac{4 - x^2}{x}$.

Additional notes

Additional notes

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Section 11.2 Techniques for Evaluating Limits

Objective: In this lesson you learned how to find limits by direct substitution and by using the dividing out and rationalizing techniques.

I. Dividing Out Technique (Pages 760–761)

> ***What you should learn***
> How to use the dividing out technique to evaluate limits of functions

The validity of the **dividing out technique** stems from ________
____
____.

The dividing out technique should be applied only when
____
____.

An **indeterminate form** is ______________________________
____
____
____
____.

When you encounter an indeterminate form by direct substitution into a rational function, you can conclude ________________
____.

Example 1: Find the following limit: $\lim\limits_{x \to 3} \dfrac{x^2 - 8x + 15}{x - 3}$.

II. Rationalizing Technique (Page 762)

> ***What you should learn***
> How to use the rationalizing technique to evaluate limits of functions

Another way to find the limits of some functions is to first rationalize the numerator. This is called the ________________
______________, which means multiplying the numerator and denominator by the conjugate of the numerator.

III. Using Technology (Page 763)

What you should learn
How to use technology to approximate limits of functions graphically and numerically

To find limits of nonalgebraic functions, ________________

____

____

____

____.

IV. One-Sided Limits (Pages 764–765)

What you should learn
How to evaluate one-sided limits of functions

Define a **one-sided limit**.

Existence of a Limit
If f is a function and c and L are real numbers, then $\lim_{x \to c} f(x) = L$

if and only if ____________________________________

____.

V. A Limit from Calculus (Page 766)

What you should learn
How to evaluate limits of difference quotients from calculus

For any x-value, the limit of a *difference quotient* is an expression of the form

____

Direct substitution into the difference quotient always produces

____.

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Section 11.3 The Tangent Line Problem

Objective: In this lesson you learned how to approximate slopes of tangent lines, use the limit definition of slope, and use derivatives to find slopes of graphs.

I. Tangent Line to a Graph (Page 770)

> ***What you should learn***
> How to understand the tangent line problem

The **tangent line** to the graph of a function f at a point $P(x_1, y_1)$ is ____
____.

To determine the rate at which a graph rises or falls at a single point, ____
__________________________________.

II. Slope of a Graph (Page 771)

> ***What you should learn***
> How to use a tangent line to approximate the slope of a graph at a point

To visually approximate the slope of a graph at a point, _______
____
____
____
____.

III. Slope and the Limit Process (Pages 772–774)

> ***What you should learn***
> How to use the limit definition of slope to find exact slopes of graphs

A **secant line** to a graph is ______________________________
____.

A **difference quotient** is ________________________________.

Give the definition of the slope of a graph.

Example 1: Use the limit process to find the slope of the graph of $f(x) = x^2 + 5$ at the point (3, − 1).

IV. The Derivative of a Function (Pages 775–776)

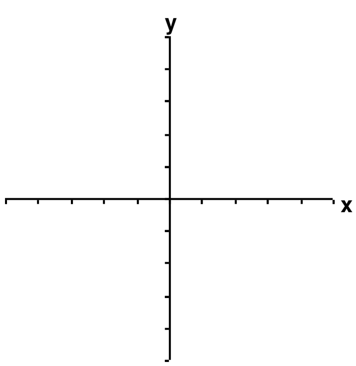
What you should learn
How to find derivatives of functions and use derivatives to find slopes of graphs

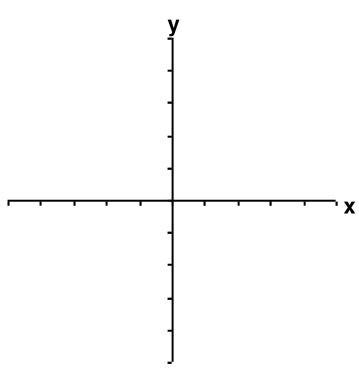
The derivative of *f* at *x* is the function derived from __________
____
____.

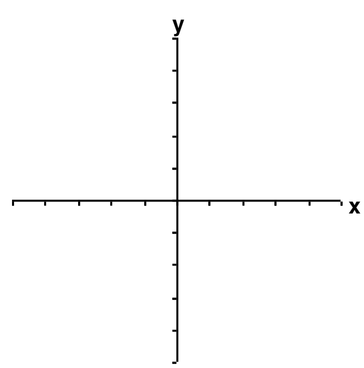
Give the formal definition of the **derivative.**

The derivative $f'(x)$ is a formula for _______________________
____.

Example 2: Find the derivative of $f(x) = 9 - 2x^2$.







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Section 11.4 Limits at Infinity and Limits of Sequences

Objective: In this lesson you learned how to evaluate limits at infinity and find limits of sequences.

I. Limits at Infinity and Horizontal Asymptotes
(Pages 780–783)

> ***What you should learn***
> How to evaluate limits of functions at infinity

Define **limits at infinity.**

To help evaluate limits at infinity, you can use the following:

If r is a positive real number, then $\lim\limits_{x\to\infty} \dfrac{1}{x^r} =$ ___________.

If x^r is defined when $x < 0$, then $\lim\limits_{x\to-\infty} \dfrac{1}{x^r} =$ ___________.

Example 1: Find the limit: $\lim\limits_{x\to\infty} \dfrac{1+5x-3x^3}{x^3}$

If $f(x)$ is a rational function and the limit of f is taken as x approaches ∞ or $-\infty$,

- When the degree of the numerator is less than the degree of the denominator, the limit is ____________________.
- When the degrees of the numerator and the denominator are equal, the limit is ____________________________ ___________________________.
- When the degree of the numerator is greater than the degree of the denominator, the limit ________________ ________________________.

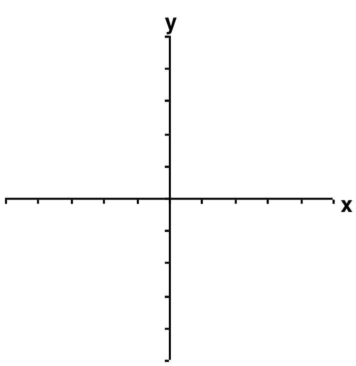
II. Limits of Sequences (Pages 784–785)

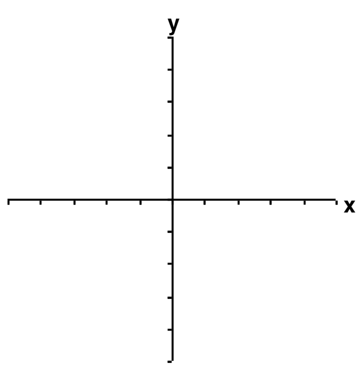
> ***What you should learn***
> How to find limits of sequences

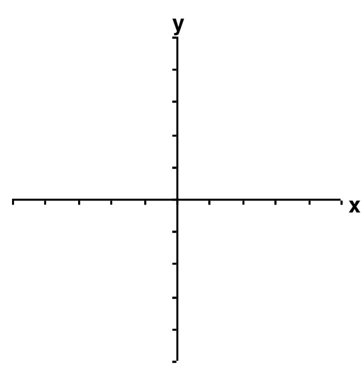
For a sequence whose nth term is a_n, as n increases without bound, if the terms of the sequence get closer and closer to a particular value L, then the sequence is said to ________________ to L. Otherwise, a sequence that does not converge is said to ________________.

Give the definition of the limit of a sequence.

Example 2: Find the limit of the sequence $a_n = \dfrac{(n-3)(4n-1)}{4-3n-n^2}$.







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Section 11.5 The Area Problem

Objective: In this lesson you learned how to find limits of summations and use them to find areas of regions bounded by graphs of functions.

I. Limits of Summations (Pages 789–791)

What you should learn
How to find limits of summations

The following summation formulas and properties are used to evaluate finite and infinite summations.

1. $\sum_{i=1}^{n} c =$ _____________
2. $\sum_{i=1}^{n} i =$ ____________________
3. $\sum_{i=1}^{n} i^2 =$ ___________________________
4. $\sum_{i=1}^{n} i^3 =$ _____________________
5. $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$
6. $\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$

To find the limit of a summation, __________________________
____
____
____.

Example 1: Find the limit of $S(n)$ as $n \to \infty$.

$$S(n) = \sum_{i=1}^{n} \frac{i-5}{n^3}$$

II. The Area Problem (Pages 792–793)

Describe the area problem.

> ***What you should learn***
> How to use rectangles to approximate and limits of summations to find areas of plane regions

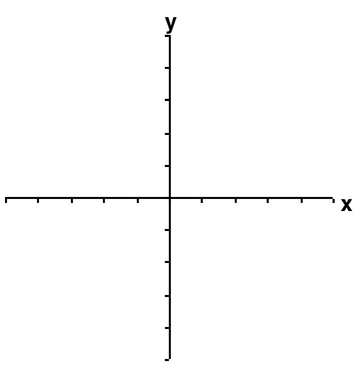
The exact **area of a plane region *R*** is given by __________

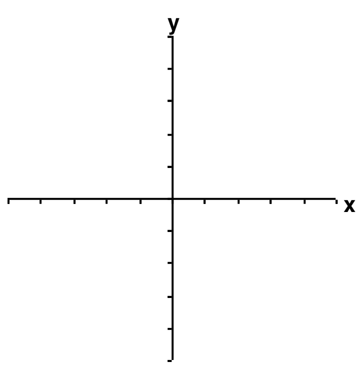
____.

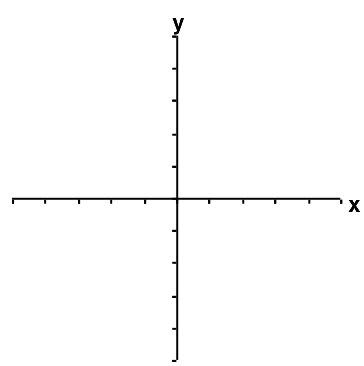
Let f be continuous and nonnegative on the interval $[a, b]$. The area A of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

____

Example 2: Find the area of the region bounded by the graph of $f(x) = (x-4)^2 + 5$ and the x-axis between $x = 3$ and $x = 6$.







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